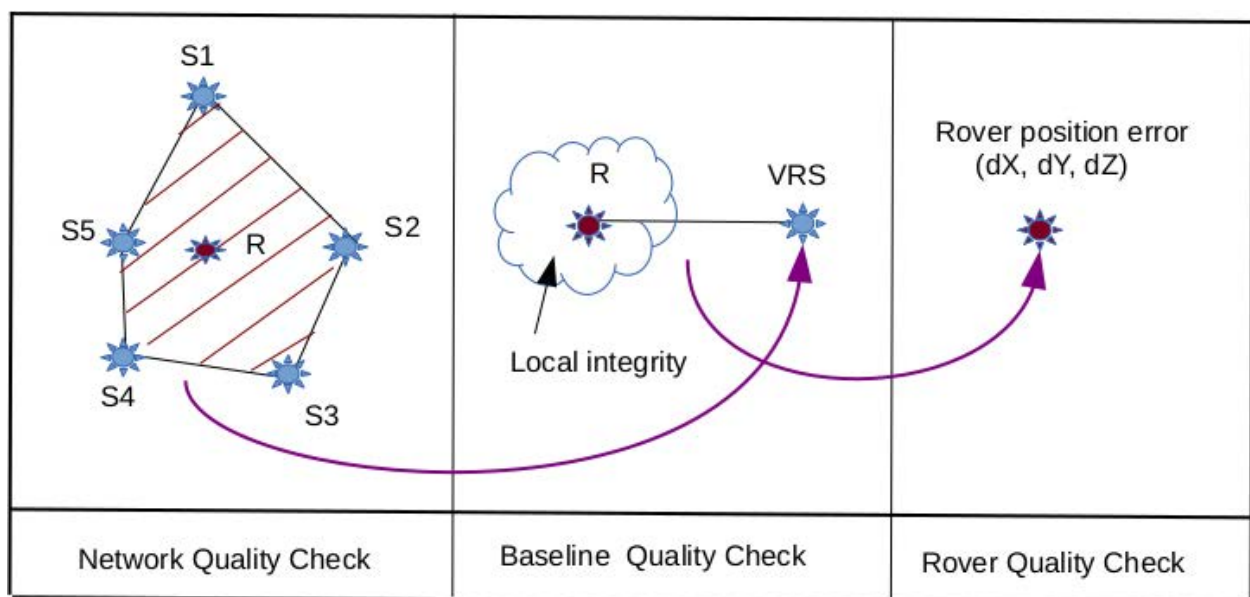




GNSS data analysis

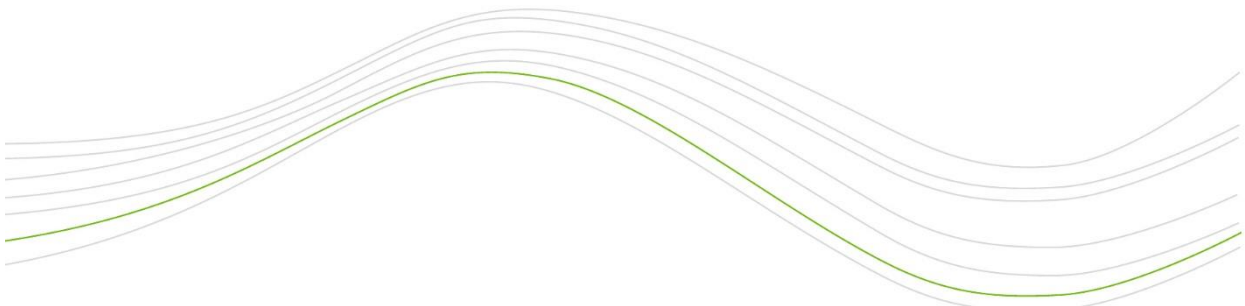
Quality assessments and integrity design

Report no. 19-04811-14



"All in all, doing data analysis is simply about calculating eigenvectors; all the science and art in it is about finding the right matrix to diagonalize."

– Jean-Paul Benzecri (1973)



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Summary: GNSS Key Performance Indicators are considered as the main building blocks of data integrity definitions. This report presents algorithms to compute such indicators. This includes the reference receiver data integrity, Signal-In-Space, atmosphere and local data integrity.

Keywords: Signal-In-Space (SIS); Key Performance Indicators (KPI); Detection, Identification and Adaptation (DIA)

Reviewer: Astrid Marie Flattum Muggerud and Knut Stanley Jacobsen

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Definitions related to GNSS

Name	Definition
Accuracy	A measure of how close an estimate of a GPS position is to the true one.
Ambiguity	During the signal acquisition by the receiver on the ground, the initial oscillator phase offset, the so-called ambiguity shall be determined and corresponds to the number of whole carrier cycles in the propagation path.
Clock Bias	The difference between the indicated clock time in the GPS receiver and true universal time (or GPS satellite time).
Clock offset	A constant difference in the time reading between two clocks, normally used to indicate a difference between two time zones.
DOP	Dilution of precision, a measure of the satellite-receiver geometry, describing factors which affects the geometric satellite distributions.
Error ellipse	Uncertainty of the measurements are described as an error ellipse, which has its center at the correct position and the size and the orientation are given by the coefficients of the ellipse equation.
Fixed effect	Unknown constant that we try to estimate from data.
GPS	A global system based on 24 satellites orbiting the earth at an altitude of approximately 20,200 km and providing precise worldwide positioning and navigation information 24 hours a day.
Kalman Filter	Recursive data processing algorithm.
Magnetic North	Direction of the magnetic north pole relative to the observers position. In other words, the direction that a compass points.

Name	Definition
Multipath	Interference caused by reflected GPS signals arriving at the receiver.
Navigation Message	The message transmitted by each GPS satellite containing system time, clock correction parameters, ionospheric delay model parameters, and the satellite's ephemeris data and health. The information is used to process GPS signals to give the user time, position, and velocity. Also known as the data message.
Outlier	Outliers are usually not just biased observations, but rather artifacts caused by filters, instruments, and other chaotic phenomena. They significantly deviate from the distribution of regular observations, and this makes them straight-forward to eliminate.
Particle Filter	Recursive data processing algorithm for non-stationary and non-Gaussian processes based on simulation.
Position	An exact unique location based on geographic coordinate system.
Precision	A measure of how close an estimate of a GPS position is to the sample mean.
Random effect	A random variable and we try to estimate the parameters that describe the distribution of this effect.
Residual	Difference between observation and prediction.
Smoothing	Backward processing algorithm.
True North	The direction of the geographic north pole from your current position. Magnetic compasses indicate north differently due to the variation between true north and magnetic north. A GPS receiver can display headings referenced to true north or magnetic north.
Troposphere	The lowest region of the atmosphere between the surface of the earth and the tropopause, characterized by decreasing temperature with increasing altitude.
WGS-84	World Geodetic System 1984, the primary map datum used by GPS. Secondary datums are computed as differences from the WGS-84 standard.

Abbreviations

Abbreviation	Description
ANUBIS	GNSS data quality control tool
BDS	BeiDou Navigation Satellite System (China)
CB_i	Check Barriers at level i
CPF	Central Processing Facilities
CPOS	Centimeter POSition based on NRTK
DIA	Detection Identification and Adaptation
DOP	Dilution Of Precision
DOY	Day Of Year
ECEF	Earth-Centered, Earth-Fixed
EGNOS	European Geostationary Navigation Overlay System
GAGAN	GPS-Aided GEO-Augmented Navigation
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
GLONASS	Globalnaja navigatsionnaja sputnikovaja sistema
GIVD	Grid Ionospheric Vertical Delay
GIVE	Grid Ionospheric Vertical Error
GSA	European GNSS Agency
HSI	Horizontal Safety Index $\left(\frac{HPE_j}{HPL_j} \right)$
IGP	Ionospheric Grid Point
ITS	Intelligent Transport System
MI	Misleading Information
NMA	Norwegian Mapping Authority
NMEA	National Marine Electronics Association
NRTK	Network RTK
NOSA	NORwegian Space Agency
NTE	Not To Exceed
PE	Position Error
PPP	Precise Point Positioning
PVT	Position, Velocity and Time.
IRNSS	Regional Navigation Satellite System (India)
KPI	Key Performance Indicator

Abbreviation	Description
QoS	Quality of Service
QZSS	Quasi-Zenith Satellite System (Japan)
LPV-200	Localizer Performance with Vertical guidance
R	Radial ephemeris error
RAIM	Receiver Autonomous Integrity Monitoring
ARAIM	Advanced RAIM
RINEX	Receiver Independent Exchange Format
RMS	Root-Mean-Square
RTCM	Radio Technical Commission for Maritime services
RTK	Real-Time Kinematic
RTIM	Real-Time ionosphere monitor
RTIS	Real-time Ionospheric scintillation
SATREF	SATellite-based REFerence system
SBAS	Satellite Based Augmentation System
SPMS	Service Performance Monitoring Support
SPP	Single Point Positioning
SoL	Safety of Life
SIS	Signal in space
SIS Anomaly	Misleading SIS information (a condition during which a healthy, trackable SIS results in WC-URE exceeding the NTE threshold)
SLM	Single Layer Model
SNR	Signal-to-noise ratio
SP3	Extended Standard Product 3 Orbit Format
SPS	Standard Positioning Service
SVN	Space Vehicle Number
TEC	Total Electron Content
TTOM	Transmission Time Of Message
URE	User Equivalent Range Error (total pseudorange inaccuracy)
URA	User Range accuracy
URA-UB	Upper Bound of URA
UEE	User equipment error (pseudorange inaccuracy due to the receiver)
URE	User range error (pseudorange inaccuracy due to the SIS)
GA-URE	Global average URE (RMS value of the URE across the portion of the globe in view of the satellite)
GA-URE ₀	Global average URE under the assumption of zero clock errors

Abbreviation	Description
I-URE	Instantaneous URE (the URE experienced by the receiver at a certain location)
SIS-URE	The same as URE
WC-URE	Worst case URE (the largest [in terms of absolute value] URE across the portion of the globe in view of the satellite)
VSI	Vertical Safety Index $\left(\frac{VPE_j}{VPL_j} \right)$
WAAS	Wide Area Augmentation System
WGS84	World Geodetic System 1984
XML	Extensible Markup Language
XTR	Xtreamer media streamer

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Introduction

“Every time I think I know what’s going on, suddenly there’s another layer of complications. I just want this damn thing solved.”

— **John Scalzi** - *The Last Colony*

This chapter provides an introduction to this report, starting off with the motivation behind the study, and discussing the specific problems that we wish to address.

1.1. Objective

The main objective of this report is to define a common analysis methodology on GNSS data from various types of measurement campaigns to provide a good factual basis. The report will be part of Norwegian Mapping Authority’s extensive series of technical reports and will serve as a guide to NMA GNSS data analysis in general.

Recently, the NMA plays a central role in society by providing a new service such as GNSS information provider with aim to increase the utilization of GNSS data in Norway to help develop the industry by creating new opportunities, services, and products. This new role can be regarded as links between technology and business. It is a very complex and resource demanding task because GNSS experiences rapid changes over time, hence today’s GNSS users may not be tomorrow’s users. Analysis includes data quality control at different levels of data processing chain, starting with data collection, archiving, monitoring, analysis, and distribution.

NMA GNSS users can be classified into three main groups.

- **Raw measurement users:** Are people using GNSS data in real-time and post-processing. In order to satisfy this group, we have to make sure that our network of reference receivers is stable and generate high quality GNSS measurements. This is station data integrity and will be treated in more details in Chap. [2](#)
- **CPOS users:** The users are primarily the companies and organizations that build and maintain the Norwegian infrastructure, e.g. those involved in road building, rail-way monitoring and maintenance, construction, surveying, and machine guiding. The CPOS users receive the corrections provided by NMA network known as CPOS and use them to compute the position at centimeter level accuracy.

To satisfy the need of these users, its recommended to continuously monitor the NMA infrastructure and make sure that the users receive reliable corrections. This is reliability analysis of network RTK and is the subject of Chap. 3

- Monitoring data users: The users are primarily the NMA users and our national and international partners (ESA, NSA, SINTEF and others). Although it seemingly that monitoring data plays a lesser role compared to the corrections and raw measurement data, the importance of this data cannot be overemphasized.
 - Validation of new concept: Data generated by NMA monitor stations are used to test a new concept, constructing key performance indicators to be used as basic required components for creating new opportunities and services.
 - Scientific studies: Mostly used to study and identify unwanted anomalies with objective to link and explain the physical effects in a logical way. The effect of using this kind of data is a real challenge, take for instance the ionospheric quality indicators known as ROTI, σ_ϕ , and S_4 . These indicators are archived and used for advanced space weather studies and they play a central role for international cooperations.
 - Quality check and data integrity: One active area of research is GNSS data integrity to the users. That is, protecting the users against the failure of the system, warning them in the shortest amount of time and with the greatest precision possible. One major problem with integrity is how to determine what can be considered as safe. Without testing different scenarios, for instance the influence of ionosphere, orbit, troposphere, multipath and baseline length on integrity computation, we cannot protect the users from errors, and safe state will be impossible to achieve. These derived KPIs will be introduced in more details in Chaps. 3 and 4.

Our aim is to carry out continuous testing, tracing, and monitoring of GNSS data at different levels of data processing. This is done in order to identify critical components that make the analysis weaker and how to deal with it properly. These activities are necessary to construct the NMA knowledge center which contributes to economic growth and society development.

1.2. NMA GNSS Analysis campaign

We list all GNSS analysis campaigns carried out by NMA. Table 1.1 lists all projects that NMA GU is involved in nationally and internationally.

Type of Analysis	Descriptions and references
C-ITS and automated transport	Appendix A.1.
Galileo OS Navigation Performance	Appendix A.2.
EGNOS Performance Monitoring	Appendix A.3.
EGNOS ionosphere model monitoring	Appendix A.4.
GNSS signal degradation in difficult environment	Appendix A.5.
GPS Subsidence data analysis of ConocoPhillips and AkerBP platforms	Appendix A.7.
SWAN project	Appendix A.9.
P3-SWE-XLI	Appendix A.10.
NSIA project	Appendix A.11.

Table 1.1.: NMA GNSS data analysis campaign

1.3. Outlooks of this report

This report is organized as five chapters and each chapter ends with a conclusion and recommendation (improvement suggestions). The first chapter introduces the main goal of this report and presents a list of specific project goals, deliverables, tasks and what we would like to achieve. The second chapter is directed to station data integrity with a user requirements definition. A Detection, Identification and Adaptation (DIA) procedures are applied to achieve this main goal by monitoring all parameters of interest. This includes monitoring the station coordinates, signal reflection from surroundings, quality check of the received signals, station clock stability and missing observations. The third chapter handles in particular GNSS parameters estimation and modeling, in addition to quality indicators determination known as Key Performance Indicators (KPI). These KPIs will be used to build a predictive model for user data integrity. The NMA Products User Interface design and implementation will be handled in Chap. 3. The main objective of this chapter is to facilitate access to NMA products without difficulties. The last chapter is the conclusion and recommendations. In addition, we will focus on the user performances and algorithms that can be used to determine the reliability of position solution. This includes

GNSS systems (GPS, GLONASS, Beidou, Galileo, GBAS, QZSS, etc.)	GNSS data acquisition CB1: Station data integrity (Chapter 2)
Data processing techniques (SPP, PPP, RTK, NRTK, PPP-RTK, PPP-AR, etc.)	Estimation & Modeling CB2: Satellite data integrity (orbits, clocks and biases) (Chapter 3)
Key Performance Indicators (KPIs)	Quality Indicators Generation CB3: Integrity main building blocks (Chapter 3)
User domain Algorithms (RAIM, CREAM, FDE, etc.)	Position data integrity CB4: Satisfaction according to the user defined requirements (Chapter 4)
NMA Analysis Software Terapos, GipsyX, Pegasus, NRTK, WHERE, RTIM, RTIS, SPRING, GAMIT, BERNESSE, Anubis, gLab, waSoft	

Figure 1.1.: GNSS data analysis segments and check barriers

Receiver Autonomous Integrity Monitoring (RAIM) family algorithms and other metrics. Finally, appendix A introduces some reports from previous works carried out by NMA, GU department.

Introduction to GNSS user requirements

“Every problem becomes very childish when once it is explained to you.”

— Arthur Conan Doyle - *A Case of Identity*

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This chapter introduces the user requirement definitions and the focus will be directed to GNSS data users. This includes station data integrity and the corresponding user requirement definitions.

2.1. Station Data Integrity

CPOS stations track signals from various navigation systems (GPS, GLONASS, Beidou, and Galileo). These signals need to be properly monitored and tested prior to their use in operational analyses. This was also the main motivation in developing a new tool which we call *"station data integrity"*.

2.1.1. Introduction

This section is devoted to the first barrier box defined in Figure 1.1, namely the station data integrity. With station data integrity, we refer to the first level in GNSS reference station networks operation. This level performs

analysis using the data from a single station to perform quality control / quality assurance procedures. That is, using the pseudorange and carrier-phase measurements to carry out the integrity check and ensure that the station clock is stable, outliers are detected and potential cycle-slips are estimated.

The aim is to detect and identify anomalies that occur at the station and cause the degradation of the user position solution. This includes the station coordinates, and disturbances of signal acquisition from GNSS satellites. Quality indicators are computed and defined by KPIs. These KPIs are stored in DIA database and can be regarded as elementary main building blocks of DIA processes.

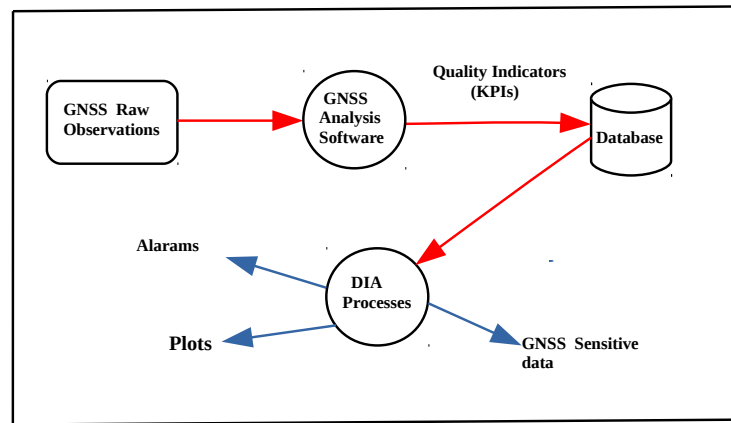


Figure 2.1.: High-level decomposition of the DIA function and external entities

Figure 2.1 shows all necessary components needed to implement the station data integrity by a DIA process. The input to GNSS data processing software packages are raw GNSS observations. The software computes the quality indicators as KPIs and store them in the database for further analysis. The DIA processes retrieve the information from database, generate alarms, plots, and notify the right responsible for further investigations. With GNSS sensitive data, we refer to the quality indicators derived from the correlated errors, namely variation of TEC, rapid variations of the TEC known as scintillation and tropospheric wet delay variation known as gradient. Our aim is to monitor the variations of the variance of these KPIs and detect the extremal events causing degradation of the user position solution. The parameters to be monitored are:

- KPI_1 : Station coordinates stability: The DIA modules check if the station coordinates are correct and stable on daily/weekly basis.
- KPI_2 : Number of cycle-slip per station: Monitored to detect the signal disturbances from surroundings for instance the multipath.

- KPI_3 : Data gap: We expect that the station shall deliver high quality data without or with minor gaps in the data.
- KPI_x : Other KPIs are defined in Section 2.2.

2.1.2. Virtual Reference Station (VRS)

Before we illustrate the importance of correctness of the station coordinates, we first introduce the VRS concept. The basic idea is then to simulate an unphysical “virtual” base station in closer proximity to the end-user than any of the physical “real” base stations, and use this virtual base station to generate synthetic RTK corrections. Figure 2.2 shows the VRS concept.

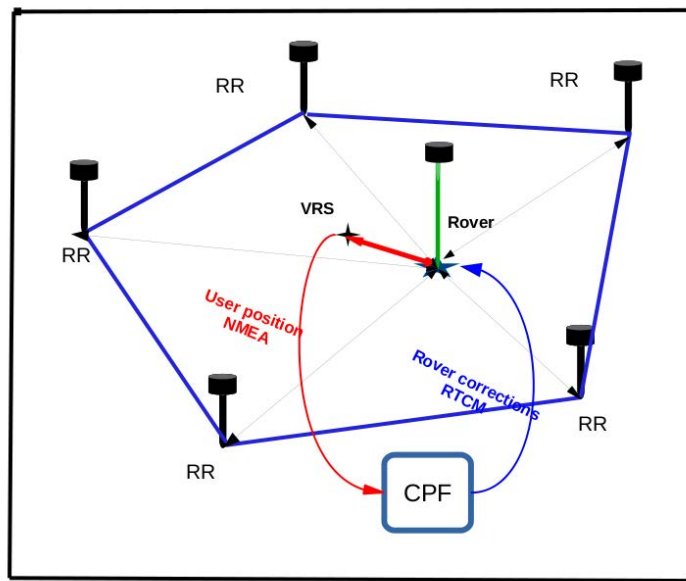


Figure 2.2.: Illustration of NRTK VRS concept. CPF stands for central processing facility, RR for reference receiver, and VRS for virtual reference station.

The procedure of the NRTK VRS method is defined as follows:

- Approximate the rover position: A user in the field sends their estimated position to the CPF via NMEA protocol. The initial position can be computed by using a code pseudo-range observables. This can be done using the SPP algorithm or alternatively using the Bancroft algorithm.
- The CPF chooses a sub-network around the rover and estimates the biases, followed by an interpolation to the rover location.

- The rover uses the corrections to compute the baseline between the true position and the approximated position using the carrier-phase double difference method. This step is known as the relative positioning. Any algorithm capable to compute the baseline is applicable.

2.1.3. Station coordinates monitoring

2.1.3.1. BERNESSE as a tool for quality check

BERNESE GNSS software package is used to compute daily and weekly station coordinates of pre-defined network. The output are station names, coordinates in ECEF and an indicator flag. The data are stored in database as table T_1 . The DIA processes retrieve the table T_1 and carries out the detection process based on some well defined test statistics. If any anomalies occur, the alarms are generated and redirected to the right person to carry out the identification process. That is, identifying the main reason behind biased coordinates. For adaptation, the station will be removed from network for a while until the problem is solved.

Here we illustrate the case when the station operates with biased coordinates. The CPOS user sends an approximation of position, then the CPOS main process selects a subnetwork in order to compute the corrections. Two scenarios occur:

- First scenario: If the user is in the vicinity of the corrupted station, the processing engine chooses that station as a virtual reference station and computes the baseline. The result is biased due to the corrupted station coordinates.
- Second scenario: Even if the user is in the vicinity of the corrupted station and we intentionally force the processing engine to compute the corrections. Under the interpolation process, the weights are assigned according to their distance from the prediction location (user location). Locations(stations) that are closer to the user location receive higher weights, and locations that are far from the user location are given lower weights. The result is less biased than the first scenarios.

Anyway, the interpolation algorithm play a central role in NRTK data processing. Poor algorithm will bias the user position solution. Interested reader is referred to an elegant paper that demonstrate the impact of the interpolation algorithms on the NRTK user accuracy [7].

2.1.3.2. Daily/Weekly station coordinates

GNSS software package BERNESSE is used to generate daily and weekly station coordinates. The output monitoring parameters are given by Table 2.1.

These parameters are stored in the DIA database on daily and weekly basis.

Items	Names	Descriptions
1	NUM	counter, increased by one for each processed station
2	STATION NAME	Four characters station name
3	X	ECEF X-coordinate in meter
4	Y	ECEF Y-coordinate in meter
5	Z	ECEF Z-coordinate in meter
6	FLAG	Flag (A: computed, W: control point)

Table 2.1.: Daily/Weekly station coordinate entities

2.1.4. DIA database

DIA Database used by DIA processes is a collection of various tables containing information about quality indicators. Tables [2.1-2.4] give the description of each DIA parameter. The DIA database can be centralized or distributed depending on design chosen under implementation.

2.1.5. Anubis as GNSS data processing tool

Anubis is a command-line tool for a quality check of data from all available GNSS constellations providing full quantitative and qualitative control with support of GNSS navigation messages, SP3 orbit products or both:

- observation statistics (counts of observations, signals, bands, satellites of GPS, GLONASS, Galileo, BeiDou, SBAS, QZSS, IRNSS)
- standard point positioning (if data and navigation messages provided for GPS, GLONASS, Galileo and BeiDou)
- pre-processing including clock-jumps and cycle-slip detection and other interruptions over all constellations, frequencies and signals
- satellite elevations and azimuths (if supported with constellation-specific navigation messages or SP3 products)
- elevation-specific histograms, satellite ascending/descending times for horizon and user elevation cut-off
- pseudo-range multipath and noise
- carrier-phase signal-to-noise ratio

- data gaps and small data pieces
- outputs in specific XTR and minimalistic standard XML formats
- other useful lists and statistics (satellite navigation data availability, satellite healthy status etc.)

The interested reader is referred to [23, 24, 25, 26].

The GNSS software library G-Nut has been developed at the Research Institute of Geodesy, Topography and Cartography since 2011. Along with the PPP applications for positioning and troposphere monitoring. The third tool recently built using the new library is called Anubis. Its initial purpose is to provides quantity and quality monitoring for multi-GNSS data stored in RINEX 2.xx (≤ 2.11) and 3.0x (≤ 3.02) formats. The Anubis is capable to handle all new emerging signals from all global navigation satellite systems and their augmentations (GPS, GLONASS, Galileo, BeiDou, SBAS and QZSS). Additionally, Anubis supports GPS, GLONASS and Galileo broadcast navigation messages, while others will be implemented soon. Supported with relevant navigation messages, Anubis performs SPP and provides GNSS data characteristics in elevation and azimuth dependencies. The pre-processing mode is used for the reconstructing observations affected by cycle slips or receiver clock jumps causing discontinuities in carrier-phase observable. A new algorithm was developed for code multipath detection supporting all signals, frequency bands and GNSS constellations. Being an open-source tool, Anubis is suitable for GNSS data providers as well as data and analysis centers for the quality and content monitoring prior to the data archiving, dissemination or a final GNSS analysis.

The output data from Anubis are given by Tables 2.2, 2.3 and 2.4 respectively and are used to build quality check and monitoring procedures of DIA processes.

2.1.5.1. Cycle-slip monitor

The cycle-slip is caused by a blockage of the satellite signal due to the man-made objects (buildings), trees, mountains, high ionospheric activities, multipath, low satellite elevation angle (El), signal processing tracking loops failure and other unknown phenomena. Anyway, the jumps must be detected and repaired in order to carry out precise positioning. To accomplish the goal, we have to construct a time series from the carrier-phase measurements, by forming a geometry-free linear combination and building a predictive model based on Auto-regressive process. The interested reader is referred to [11, p. 101].

The parameter of interest here is the number of cycle-slip observed for each GNSS signal tracked at the station of CPOS network. Forming a time-series and constructing a test-statistics to test for trend. Denoting by N the number of cycle-slip in normal situation, and N_{obs} is the observed one in sliding window with length of x days, then the hypothesis testing reads:

$$\begin{aligned} H_0 &: N = N_{obs} \quad , \text{ No trend in the data} \\ H_1 &: N \neq N_{obs} \quad , \text{ Presence of trend in the data} \end{aligned} \quad (2.1)$$

Under H_0 , no trend is present in time-series, while the alternative hypothesis H_1 , corresponding to $N \neq N_{obs}$ means that there is trend (large-scale variations). This test gives us the indication for multipath caused by surrounding (trees). For small-scale variations are caused by high ionospheric activities, similar test can be constructed with smaller sliding window length.

Details on how to construct such test statistics will be discussed with panel participants before implementation.

Forming time-series to monitor the cycle-slip can be carried out by a procedure involving the following parameters generated by Anubis and provided by the following tables:

- $o/slps$ parameter provided by Table 2.2.
- $csTot$ and $nSlp$ parameters provided by Table 2.3.
- CS_{Tot} and $CSlips$ parameters provided by Table 2.4.

2.1.5.2. Data gap monitor

Data gap is a serious problem in GNSS data analysis and introduces discontinuities in observation and make it difficult to resolve the ambiguities.

2.1.5.3. GNSS overall summary

Description of columns in overall summary is following:

Items	Names	Descriptions
1	Hours_	data length in hours (estimated from total number of epochs and sampling rate)
2	Sample	data sampling interval (estimated from elevation histogram)
3	MinEle	data minimum elevation angle observed
4	#Expt	number of expected observations above the horizon (carrier phases with maximum observations over GNSS counted)
5	#Have	number of existing observations above the horizon (carrier phases with maximum observations over GNSS counted)
6	%Ratio	ratio of existing and expected observations above the horizon (calculated from above values)
7	o/slps	number of observations per cycle-slip
8	woElev	number of observations with available elevation (no sat. position available)
9	Expt $\geq$ 10	number of expected observations above user mask (carrier phases with maximum observations over GNSS counted)
10	Have $\geq$ 10	number of existing observations above user mask (carrier phases with maximum observations over GNSS counted)
11	%Rt $\geq$ 10	ratio of existing and expected observations above user mask (calculated from above values)

Table 2.2.: GNSS constellation summary

2.1.5.4. GNSS-specific summary

Description of columns in GNSS-specific summary is following:

Items	Names	Descriptions
1	ExpEp	number of expected epochs (uses pre-defined data interval and sampling or suggests from data if not available)
2	HavEp	number of available epochs (really counted epochs)
3	UseEp	number of usable epochs (criterion is 4 satellites in the epoch with dual-frequency data in each GNSS constellation)
4	xCoEp	number of epochs with less than 4 satellites providing dual-frequency pseudorange observations
5	xPhEp	number of epochs with less than 4 satellites providing dual-frequency carrier-phase observations
6	xCoSv	number of single-frequency pseudorange data only
7	xPhSv	number of single-frequency carrier-phase data only
8	csTot	number of total phase cycle-slips or interruptions (in general means request to setup a new initial phase ambiguity)
9	csEpo	number of interruptions due to missing epoch (counts all previously observed satellite)
10	csSat	number of interruptions due to missing satellite at a specific epoch (where it is expected)
11	csSig	number of interruptions due to missing signal from the satellite (others are available)
12	nSlp	number of identified phase cycle-slips when continuous tracking is available for specific satellite and signal
13	nJmp	number of identified receiver clock jumps
14	nGap	number of data total gaps (according to the setting $\text{int}_{gap} = 600$ in sec)
15	nPcs	number of small data pieces (according to the setting $\text{int}_{pcs} = 1800$ in sec)
16	mpX	code multipath moving average RMS [cm] for the 1st..8th band, (mean value over all available signals at the 1st..8th band)

Table 2.3.: GNSS-specific summary

2.1.5.5. Observation types summary

High verbosity provides results of statistics of individual observation types of all available GNSS constellations (see listing below). If navigation messages for specific constellations are available (over the whole QC period!), all

intervals of individual satellite visibility can be theoretically calculated and expected observations estimated. Histograms of observations available in columns 0[%] and 0-10, 10-20, .., 80-90 elevations are additionally provided. However, these are actually time-consuming procedures and rather tricky if not all relevant navigation messages are available. Thus the features are supported only experimentally with an extra high verbosity via setting $sec_{sum} = "9"$.

Items	Names	Descriptions
1	nSat	number of observed satellites
2	ExpObs	number of expected observations (an estimate)
3	obs[%]	existing observations w.r.t. expected
4	nor[%]	existing observations w.r.t. expected, but normalized with rate of existing satellites vs. total expected in the constellation
5	CS _{Tot}	number of total phase cycle-slips and phase interruptions
6	CS _{Epo}	number of carrier-phase interruptions due to missing epoch (counted for all previously observed satellites and signals)
7	CS _{Sat}	number of carrier-phase interruptions due to temporary unavailable satellites
8	CS _{Sig}	number of carrier-phase interruptions due to temporary unavailable signals
9	CSlips	number of identified real phase cycle-slips in a continuous phase tracking

Table 2.4.: Observation types of all available GNSS constellations

2.1.6. Conclusion and remarks

CPOS stations track signals from various navigation systems (GPS, GLONASS, BDS, and Galileo). These signals need to be properly monitored and tested prior to their use in operational analyses. This was also the main motivation in developing a new tool which we call station data integrity.

We have shown that the station data integrity can be achieved by a DIA procedure. In addition, the panel participants agreed that the GNSS data processing softwares (BERNESE, GipsyX, and Anubis) are indispensable tools to compute the quality indicators that serve as KPIs.

2.2. Station data integrity: User Requirement Definition

This section focuses on station data integrity user requirements definition by DIA procedures. To ease the mapping between user and system requirements, it is necessary to define some coding pattern. Our suggestions are (DA: Data Analysis, SDI: Station Data Integrity, URD: User Requirement Definition).

2.2.1. GNSS_DA_SDI_URD_0010: Choosing station location

Choosing location for receiver placement is important task and must be carried out carefully. Some guidelines are:

1. Grounding and leveling tests.
2. Lightning tests: Shall be used to detect if the area is exposed to lightning.
3. Spectrum analyzer: Shall be used to detect any interfering radio signals on GNSS frequencies.
4. Good horizon for receiving satellite signals.
5. Accessibility: It is important that the station is easily accessible to minimize response time for the maintenance crew.
6. Antenna mast/mount foundation stability

Directives and procedures are defined in appendix [A.12](#) with reference documents.

2.2.2. GNSS_DA_SDI_URD_0020: GNSS equipment

Station shall be able to track all configured GNSS signals. This includes the GPS, GLONASS, Galileo, and BDS navigation systems. The station shall be equipped with:

- Multi-GNSS receiver:
 - Capable of tracking all configured GNSS signals.
 - Triple-frequency with minimum 500 frequency channels for tracking of GNSS signals.
 - At least 1 Hz data sampling, but preferably higher sampling rates (e.g. 50 Hz).

- Automatic restart with intact configuration after e.g. power failure.
- Parallel internal logging of raw data with support of RINEX format. Raw data should be logged at least over 30 days with 1 Hz data rate in proprietary format of receiver provider.
- Web-interface for remote configuration, status view, restart and upgrading the firmware.
- FTP server and FTP client to push logged data to control center.
- Real-time data streaming via RTCM (version 3.1 or 3.2).
- At least one port RJ45 with 10/100 Mbit.
- Minimum 2 TCP-ports which may be configured as client.
- Configuration of receivers:
 - * Smoothing, clock steering and multipath rejection should not be applied.
 - * Elevation cut-off angle should be 0 degree.
- Receiver antenna:
 - Use of geodetic antennas, which reduce the multipath effect from low elevation angles (preferable choke-ring antenna) and can track all GNSS frequencies and signals.
 - Depending on used antenna it should be taken into account to use a radome to protect antenna from extreme salt, dust, solar radiation or pollution. Snow lying on top of the antenna or birds sitting on the antenna can have an impact on antenna phase center stability. Therefore conical radomes should be preferred to mitigate that snow is lying or birds sitting on top of the antenna. Further discussion necessary if conical radomes are the best choice.
 - Antenna should be calibrated for all available GNSS frequencies together with radome.
 - Operating antenna temperature should be between -40 to +55 °C.
 - Antenna should be waterproof.
- Antenna cables: Check specification of providers related to antenna cables, splitters or antenna cable connectors to ensure necessary signal strength for transmitting GNSS signals from antenna to receiver.
- External clocks: If an atomic clock (H-maser, Rubidium, Cesium) is available at a station, then this should be used as external frequency input for receiver.

2.2.3. GNSS_DA_SDI_URD_0030: Station coordinates monitoring

Weekly and daily station coordinates shall be monitored. The alarm will be delivered in cases of detection of extremal discontinuities. The threshold values are defined for ENU coordinates unity of length.

1. Daily solution

- North component threshold value $N_{th} \geq 2.0$ cm.
- East component threshold value $E_{th} \geq 2.0$ cm.
- Vertical component threshold value $U_{th} \geq 4.0$ cm.
- Maximum number of days to generate alarms ($D_A = 5$) days.

2. Weekly solution

- North component threshold value $N_{th} \geq 1.0$ cm.
- East component threshold value $E_{th} \geq 1.0$ cm.
- Vertical component threshold value $U_{th} \geq 2.0$ cm.
- Maximum number of days to generate alarms ($D_A = 3$) days.

2.2.3.1. GNSS_DA_SDI_URD_0040: Multipath detection from surrendering

If the station is exposed to multipath, the alarms must be generated.

- The parameter *mpx* from Table 2.3 is the code multipath moving average RMS, expressed in centimeter. Can be used to detect multipath by forming a time-series and studying the variation over time period (Δt). The interested reader is referred to [25, chap. 4].
- Maximum number of days to generate alarms ($D_A = 5$) days.

2.2.4. GNSS_DA_SDI_URD_0050: Monitoring the number of cycle-slip

The number of cycle-slip occurrences at the station shall be monitored by forming a time-series over a time period (Δt) and studying the variations. Extremal events shall be reported by generating alarms. The parameter CS_{tot} from Table 2.4 defined as the number of total phase cycle-slips and phase interruptions can be used to detect anomalies at the station.

2.2.4.1. GNSS_DA_SDI_URD_0060: Monitoring the quality of the received signals

Carrier-phase signal-to-noise ratio (S2NR) shall be monitored by forming a time-series over a time period (Δt) and studying the variations. Extremal events shall be reported by generating alarms.

The S2NR represents a generic term for signal quality and is defined as the ratio of signal power S in watts (W) to noise power N in W, measured at the same time and place in a circuit. The signal and noise power can be estimated during the synchronization (or correlation) between the received and replica signals. The main part of noise originates from the receiver electronics (e.g., thermal noise created by the inevitable motion of electrons within any conductor or semiconductor) and the electromagnetic radiation from the sky, ground and objects in the antenna's vicinity. Thermal noise is generally assumed to be uncorrelated (white) noise with a Gaussian distribution.

S2NR can be used to enable a more realistic assessment of observation quality under non-ideal observational conditions. That is, to improve the GNSS stochastic model by implementing a S2NR-based observation weighting model.

2.2.5. GNSS_DA_SDI_URD_0070: Missing observations

If the number of missing epochs from a station exceeds 5 % of the total expected epochs, the alarm must be generated.

2.2.6. GNSS_DA_SDI_URD_0080: Data quality control file

Data quality control file shall be generated from alarms and delivered as a companion together with raw observations to the user.

2.2.7. GNSS_DA_SDI_URD_0090: Station Clock Stability

If the station clock is unstable, the alarm must be generated regardless of the algorithm applied to monitor the clock stability. The maximum number of days to generate alarms ($D_A = 3$) days.

2.2.7.1. GNSS_DA_SDI_URD_0100: Detection response time

Under any anomalies at the station, alarms must be delivered in a period of time less than 5 days.

2.2.8. GNSS_DA_SDI_URD_0110: Identification response time

The identification of the main source of the problem causing alarms must be determined in a period of time within 7 days.

2.2.9. GNSS_DA_SDI_URD_0120: Adaptation response time

Adaptation process of the problem causing alarms must be decided in a period of time within 7 days.

2.2.10. GNSS_DA_SDI_URD_0130: Raw measurement delay monitoring

Monitoring the time delay of the output measurement from a reference station in real-time. Alarm shall be generated in case of the time delay exceeds the threshold D_{th} .

2.2.11. Conclusion and remarks

The aim is to obtain a reliable reference station that is capable of collecting and transmitting high quality GNSS raw observations to the center processing facilities. To accomplish this goal, its recommended to implement all requirements defined above and continuously monitor all parameters of interest. This step is the required and its resource saving. Detecting anomalies in early stage of data processing chain is absolutely necessary.

GNSS integrity main building blocks definition

“Why does he insist that we must have a diagnostics ? Some things are not meant to be known by man.”

— **Susanna Gregory** - *An Unholy Alliance*

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Section 3.2

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Section 3.3

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The aim of this chapter is the definition of the GNSS data integrity main elementary building blocks that can be used to alarm the user in the field in case of system malfunction. Detecting anomalies in early stage of GNSS data processing will avoid the user performance degradation and will ameliorate the user position accuracy. The starting point of our strategy is to decompose the user range errors based on GNSS error sources into different main segments and at each segment, we try to construct some quality performance indicators that can mapped to the user position accuracy. It is well known that the user position error is dependent on the geometry of receiver-satellite vectors and the errors in the pseudorange measurement. The latter, total pseudorange measurement inaccuracy, is often referred to as user equivalent range error (UERE) in the field of satellite navigation. The three major sources of UERE are:

- Control and space segment errors: Pseudorange inaccuracy due to the Signal-In-Space (SIS), also referred to as SIS URE, including satellite ephemeris errors, satellite clock errors, satellite antenna variations, and signal imperfections. This topic is the subject of Subsection 3.2.
- Propagation errors: Pseudorange inaccuracy due to atmospheric propagation modeling, including ionosphere and troposphere delays compensation errors. The topic will be treated in Subsection 3.3.
- Measurement errors: Pseudorange inaccuracy due to the receiver and the environment, also referred to as user equipment error (UEE), including multipath and receiver noise. This topic is similar to station data integrity handled in Chap. 2 and is the subject of Subsection 3.5.

For each segment defined above, quality performance indicators will be constructed. The details are given in the following sections.

3.1. Station data integrity

Reference station stability is the primary key to achieve a high quality GNSS measurements. This includes station clock stability, multipath, coordinate checks and high quality antenna and GNSS receiver. The topic was treated in Chap. 2 and will not be repeated here.

3.2. SIS data integrity

In order to guarantee the integrity of GNSS users, detecting and excluding SIS anomalies is indispensable. SIS is composed of two main components, the orbit and clock errors. Algorithms and data processing methods to detect and monitor the SIS range errors for GPS, GLONASS, Galileo and BeiDou will be provided. This section is organized as follow; Subsection 3.2.1 introduces the GNSS SIS URE. Anomaly detection and monitoring for GPS navigation system is the subject of Subsection 3.2.3.1. GLONASS will be treated in Subsection 3.2.3.2, while Galileo and BeiDou are handled in Subsections 3.2.3.3 and 3.2.3.4 respectively. Finally, Subsection 3.2.6 will examine the assumptions about the distribution of nominal UREs and the correlation among UREs of different satellites.

3.2.1. Representation of SIS URE

Signals are the interface between the user and navigation system. Full description about these signals are provided by the Interface Control Docu-

ment (ICD) and it's recommended to be used as a primary reference. For GPS [35], GLONASS [36], Galileo [37], and BeiDou [41].

Navigation messages are computed by navigation system control segment and uploaded to satellites. Satellites send these messages to the user to compute the satellite position, velocity, acceleration and clock parameters. Our aim is to control the error provided by satellite orbit and clock. Usually, we compare the broadcast message with a reference signal provided by GNSS analysis centers for instance IGS, known as precise ephemeris and clock. Quality indicators are computed based on these broadcast and precise orbits and clock products. The SIS URE is considered as a quality indicator candidate. SIS URE is dominated by ephemeris and clock errors because antenna variations and signal imperfections are usually at centimeter or millimeter level. SIS URE is a pure spatial process and its value is different for users at different locations on Earth. Neglecting antenna variations and signal imperfections, the instantaneous SIS URE (I-URE) for a user at the location $\mathbf{u}$ reads:

$$\text{I-URE} = (\mathbf{r}^t - \mathbf{r}) \cdot \frac{\mathbf{r} - \mathbf{u}}{\|\mathbf{r} - \mathbf{u}\|} - c(\mathbf{b}^t - \mathbf{b}) \quad (3.1)$$

where $(\mathbf{x} \cdot \mathbf{x})$ is the vector dot product, $\|\mathbf{x}\| = \sqrt{\mathbf{x}^t \cdot \mathbf{x}}$ is the vector norm, c is the speed of light in vacuum, $\mathbf{r}$ is the actual satellite position, $\mathbf{b}$ is the actual satellite clock bias, $\mathbf{r}^t$ is the satellite position calculated from the broadcast ephemeris, and $\mathbf{b}^t$ is the satellite clock bias calculated from the broadcast clock. Since I-URE is spatially dependent, two spatially-independent measures are defined.

- Global average SIS URE (GA-URE): Is defined as the root mean square (RMS) URE across the portion of the globe in view of the satellite (see [30] for the derivation), generally a performance metric of "SIS accuracy".

$$\text{GA-URE} \equiv \begin{cases} \sqrt{(0.98R - T)^2 + (A^2 + C^2)/49} & \text{for GPS} \quad (3.2a) \\ \sqrt{(0.98R - T)^2 + (A^2 + C^2)/61.2} & \text{for Galileo} \quad (3.2b) \\ \sqrt{(S_1 * R - T)^2 + (A^2 + C^2)/S_2} & \text{for BeiDou} \quad (3.2c) \\ \sqrt{(0.98R - T)^2 + (A^2 + C^2)/45} & \text{for GLONASS} \quad (3.2d) \end{cases}$$

where R is the radial ephemeris error, A is the along-track ephemeris error, C is the cross-track ephemeris error, and T is the clock error.

- Noting that for BDS, there are three orbits of satellites, namely MEO, IGSO and GEO. Satellites C01 to C05 are GEO satellites, satellites C06 to C10 are IGSO satellites, and satellites C11 to C14 are MEO satellites. Because of the differences in satellite observation geometry structure, the computational models for the SIS UREs of the three orbits are different and depends on the values of S_1 and S_2 , respectively. For BDS MEO, the values of S_1 and S_2 are .98 and 54, and for BDS IGSO/GEO, the values of S_1 and S_2 are .99 and 127.
- Constants defined in Eqs. (3.2) are known as weight factors. The computation is based on elevation mask of 0 degrees. Different values can be obtained for different elevation masks.

Derivation of Eqs. (3.2) is given in Section 3.2.5

- Worst case SIS URE (WC-URE): Is defined as the largest URE across the portion of the globe in view of the satellite, generally a performance metric of "*SIS integrity*".

$$\text{WC-URE} = \max_{|\theta| \leq \nu} (R \cos \theta - T + \sqrt{(A^2 + C^2)} \sin \theta) \quad (3.3)$$

where $\max(\cdot)$ maximizes and returns the value of the expression, θ is the off-nadir angle, and the maximum off-nadir angle $\nu = 13.85, 14.48, 12.43$, and 13.21 for GPS, GLONASS, Galileo and BeiDou. The off-nadir angle is defined as the angle between two line-of-sights, the first one connecting satellite to the center of the Earth and the second is from the satellite to the user receiver.

WC-URE can be computed either numerically from UREs or analytically from (R, A, C, T) using the algorithms described in 3.2.3.1.

3.2.2. GNSS anomalies detection and validation

Finding GNSS anomalies consists of the following two main steps:

1. Compute the UREs of each satellite at each epoch.
2. Check the WC-URE against NTE threshold T_h .

There are two ways to calculate the URE:

$$\text{URE} = \text{projected ephemeris error} + \text{clock error} \quad (3.4)$$

$$= \text{UERE} - \text{propagation errors} - \text{UEE} \quad (3.5)$$

The approach (3.4) is based on the fact that the URE is dominated by the ephemeris and clock errors. To calculate the ephemeris and clock errors, we need to compare broadcast ephemeris and clock with the ground truth, the actual satellite position and clock. A good candidate of ground truth is the precise ephemeris and clock generated by some organizations, such as IGS, which routinely post-process observation data. In order to accomplish the main goal mentioned above, a strategy to process the SIS information is needed. Two approaches are considered, the "*bottom-up approach*" known also as "*space approach* [Eq.(3.4)]" because involves only the information related to the satellite, and "*top-down approach* [Eq.(3.5)]" known as ground approach. The interested reader on this subject is referred to [33].

In order to monitor the GNSS anomalies, the DIA procedure is applicable and consist of the following phases:

1. GNSS anomaly detection by using the "*space approach*".
2. GNSS anomaly validation by using the "*ground approach*".

3.2.3. GNSS anomalies monitoring

GNSS SIS anomaly detection is a complex process that requires careful planning to process and examine different types of data with different sampling rate and intervals. Algorithms subscribed in [34] are applicable and will be used for derivation of quality indicators. DIA approach will be used to monitor GNSS SIS anomalies.

3.2.3.1. GPS anomalies monitoring

3.2.3.1.1. GPS anomaly detection: A grid model is chosen to organize, model, and compute the SIS quality indicators. The method is expected to be accurate as long as the grid is dense enough. A dense grid, however, leads to a heavy computational burden. Therefore it would be nice to balance between computation tasks and grid density. Satellite position, velocity, acceleration and clock parameters are computed using the algorithms in [35] and we expect that these messages are validated. Next, a sliding window is chosen to process the data. For each $t \in [5, 15]$ minutes epoch that coincides with precise ephemeris/clocks, the latest transmitted broadcast ephemeris/clock is chosen to calculate the WC-URE. In case of disjoint epochs between precise and broadcast messages, the interpolation algorithms such as Lagrange/Neville can be used. The interested reader on this topic is referred to [44]. The WC-URE can be calculated either numerically or analytically. One option is to use the brute-force numerical method and proceed as follow:

1. Define a grid over Earth with size 2.5 X 2.5 degrees.
2. For each satellite at each epoch t .
 - For each node of the grid and for each configured receiver, compute I-URE.
 - Find the I-URE with the greatest absolute value.

As mentioned above that the grid density leads to a heavy computational burden, the second option is to apply the geometric method and is preferred. Figure 3.1 shows the concept, and we assume that the Earth is a perfect sphere and the calculation of the WC-URE is achieved by using the following algorithm.

1. Find the plane that contains the center of Earth and the ephemeris error vector v .
2. Find the angle α using the inner product, and β using the law of sines (please note that $\gamma = 90^\circ + \text{mask angle}$);
3. Find the maximum and the minimum projection of v in the cone:

$$\begin{aligned}
l_{max} &= \max_{|\theta| \leq \beta} |v| \cos(\alpha + \theta), \\
l_{min} &= \min_{|\theta| \leq \beta} |v| \cos(\alpha + \theta),
\end{aligned} \tag{3.6}$$

4. Find $l_{max} - cB$ and $l_{min} - cB$, where c is the speed of light and B is the satellite clock error. The one with the greatest absolute value is the WC-URE.

The geometric method outperforms the brute-force method in terms of the computational complexity. A flaw of the geometric method is the assumption of a perfect sphere for Earth. Fortunately, the resulting approximation error is not more than 0.6%, perfectly acceptable for the purpose of this study. Finally, a GPS SIS anomaly is claimed when all the following conditions are fulfilled.

- The WC-URE exceeds the NTE threshold.
 - GPS SPS PS 2001: $\max\{30 \text{ m}, 4.42 \times \text{URA-UB}\}$.
 - GPS SPS PS 2008: $4.42 \times \text{URA-UB}$.
- The broadcast navigation message is healthy, i.e.,
 - The RINEX field SV health is 0, and
 - The URA-UB ≤ 48 meters,

- The broadcast navigation message is in its fit interval, i.e., $t = t - \text{TTOM} \leq 4$ hours;
- The precise ephemeris/clock is available and healthy.

3.2.3.1.2. GPS anomaly Identification: Anomalies detection are based on predefined threshold values, derived from observed data. As usual, often we build a model or predictive model that is derived from normal (non-anomalous) training data, then perform detection based on the prediction error for observed data. For adaptation, we ignore or we activate an external algorithm to alarm such events and register the type of alarms.

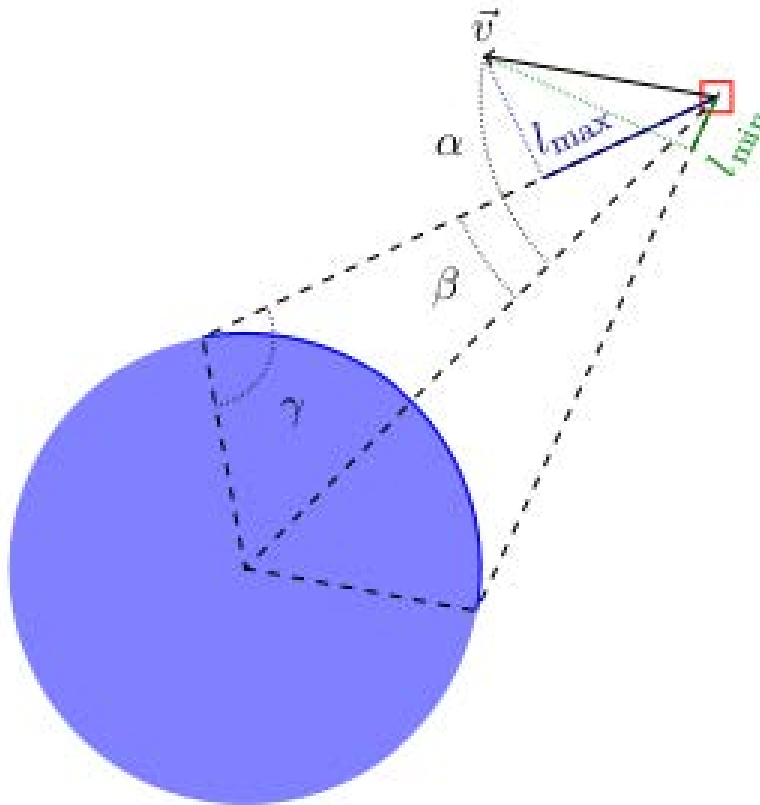


Figure 3.1.: Geometric method for calculating the WC-URE

3.2.3.2. GLONASS anomalies monitoring

Our aim is to extend SIS anomaly monitoring to GLONASS as done in previous subsection for GPS navigation system. To start with, the GLONASS

navigation system is different from GPS in many ways. The differences include the ephemeris format, navigation message update frequency, reference frame, and reference time.

3.2.3.3. Galileo anomalies monitoring

Intensive studies have been carried out recently to test the European navigation system, Galileo and to make sure that it is functioning as expected compared to the GPS. A short list of relevant literature are given herein summarizing the work done so far in this field. Galileo SIS range errors are described in more details on [40] for algorithm determinations. Analysis of Galileo SIS anomalies were calculated over two years (2017–2018) [39] by using the broadcast ephemeris provided by the MGEX, and the precise ephemeris provided by GFZ. The study shows that the main cause of the detected anomalies are directly related to the satellite clock error, which causes the SIS URE to reach tens or hundreds of meters. The anomalies in 2018 were significantly reduced, compared to those in 2017, indicating an improvement in the performance of the SIS. A preliminary analysis of Galileo F/NAV broadcast Clock and Ephemeris is performed in that paper with 43 months of data. Based on these observation results, the Galileo Signal-in-Space is assessed, and the probability of satellite failure is estimated. The Galileo nominal ranging accuracy is also characterized. Results for GPS satellites are included and to compare Galileo performances with a consolidated constellation. The analysis over the last three-year window shows promising results and the details are given in [38].

3.2.3.4. BeiDou anomalies detection and monitoring

The occurrence of SIS anomalies from the BeiDou navigation satellite System may be more frequent than for the GPS, because the BDS is new compared to the classical navigation system, the GPS. In order to guarantee the integrity of BeiDou users, detecting and excluding SIS anomalies is indispensable. The traditional method through the comparison between the final precision ephemeris and the broadcast ephemeris is limited by the issue of long latency of precision ephemeris release.

Last decade, an intensive tests were carried out for the BeiDou navigation system and we will mention some of them that directly linked to SIS. Starting with a paper provided by Cheng et al. [42], estimates in real-time, the Instantaneous SIS URE (I-URE) by using the Kalman filtering-based carrier-smoothed-code to detect and exclude BDS SIS anomalies, in which the threshold for BDS I-URE anomaly detection are obtained from the integrity requirement. The results based on 1 Hz data from ground observations

show that the proposed method has an estimation accuracy of 1.1 meter for BSD I-URE. The proposed method can effectively detect the SIS anomalies caused by either orbit faults or clock faults. Another paper provided by Wang et al. [43] addresses the impact of SIS User Range Errors (SIS URE) on ARAIM availability and integrity.

3.2.4. GNSS anomalies validation

In order to evaluate and verify the proposed algorithms, the chosen strategy is based on comparison test, that is, comparing the anomalies found using our validated navigation messages with those found using for instance the IGS daily global combined broadcast navigation messages. To carry out successfully the comparison test, the following parameters shall be considered.

- Test data: At least five years of GNSS data are required to test and evaluate the proposed algorithms.
- Reference signal. For this purpose, the NGA APC precise ephemerides/clocks can be employed as the truth reference.
- Threshold values: The SPS PS 2008 NTE threshold [30] shall be used to detect anomalies.
- Classification: All the potential SIS anomalies found in processing period will be classified as follow:
 - True anomalies: Correct decision, which indicates that the detection algorithm does not introduce any false anomalies.
 - False anomalies: False decision.

3.2.5. Computation of global average URE

The aim of this section is the derivation of Eq. (3.2) analytically. For an arbitrary ephemeris and clock errors, I-URE performance indicator is dependent on the location of receiver. The global average SIS-URE (GA-URE) is defined as the root mean square (RMS) URE across the portion of the globe in view of the satellite. In order to assess the SIS accuracy performance, the GA-URE is selected because it is spatially independent. Assuming the Earth is a perfect sphere with a normalized radius 1. Without loss of generality, assume the actual satellite position is $r = (0, 0, d)$ in ECEF, where d is the normalized distance between the satellite and the center of Earth. In addition, assume the satellite instantaneous velocity is parallel to the x-axis. According to

Eq. (3.1), the I-URE of the receiver at latitude θ , longitude ϕ , and height 0 is given by:

$$\begin{aligned}
\text{I-URE} &= (\mathbf{r}^t - \mathbf{r}) \cdot \frac{\mathbf{r} - \mathbf{u}}{\|\mathbf{r} - \mathbf{u}\|} - c(\mathbf{b}^t - \mathbf{b}) \\
&= [(A, C, d + R) - (0, 0, d)] \cdot \frac{(0, 0, d) - [\cos \theta \cos \phi, \cos \theta \sin \phi, \sin(\theta)]}{\|(0, 0, d) - [\cos \theta \cos \phi, \cos \theta \sin \phi, \sin \theta]\|} - T \\
&= [(A, C, R) - (0, 0, d)] \cdot \frac{[-\cos \theta \cos \phi, -\cos \theta \sin \phi, d - \sin \theta]}{\sqrt{(1 + d^2 - 2d \sin(\theta))}} - T \\
&= \frac{-A \cos \theta \cos \phi - C \cos \theta \sin \phi + R(d - \sin \theta)}{\sqrt{(1 + d^2 - 2d \sin \theta)}} - T
\end{aligned} \tag{3.7}$$

where R is the radial ephemeris error, A is the alongtrack ephemeris error, C is the crosstrack ephemeris error, and T is the clock error. The computation of GA-URE reads:

$$\text{GA-URE}^2 = \frac{1}{S} \int_v^{\pi/2} \int_{-\pi}^{\pi} \|\text{I-URE}\|^2 \cos \theta d\theta d\phi \tag{3.8}$$

where $S = 2\pi(1 - \sin v)$ is the area of satellite coverage, and ξ is the latitude of the edge of the satellite coverage. The evaluation of the integral given by Eq. (3.8) analytically is difficult and approximation numerical results obtained using ([44], [45], [46]).

For verification/validation of approximation the Eq. (3.8), we are going to consider well known navigation systems, including the GPS, GLONASS, Galileo, and BeiDou.

- GPS

Assuming a zero mask angle and using $r = 4.175$, $\xi = 13.85^\circ$, the corresponding GA-URE reads:

$$\begin{aligned}
\text{GA-URE}_{\text{GPS}}^2 &= .9593R^2 - 1.959RT + T^2 + .02034(A^2 + C^2) \\
&\approx (.9593R - T)^2 + \frac{A^2 + C^2}{49.2}
\end{aligned} \tag{3.9}$$

- GLONASS

Using $r = 3.998$, $\xi = 14.48^\circ$, the corresponding GA-URE reads:

$$\begin{aligned}
\text{GA-URE}_{\text{GLO}}^2 &= .9555R^2 - 1.955RT + T^2 + .02224(A^2 + C^2) \\
&\approx (.977R - T)^2 + \frac{A^2 + C^2}{45.0}
\end{aligned} \tag{3.10}$$

- Galileo

Using $r = 4.645$, $\xi = 12.43^\circ$, the corresponding GA-URE reads:

$$\begin{aligned} \text{GA-URE}_{\text{GAL}}^2 &= .967R^2 - 1.967RT + T^2 + .001632(A^2 + C^2) \\ &\approx (.984R - T)^2 + \frac{A^2 + C^2}{61.2} \end{aligned} \quad (3.11)$$

- BeiDou

Using $r = 4.375$, $\xi = 13.21^\circ$, the corresponding GA-URE reads:

$$\begin{aligned} \text{GA-URE}_{\text{BDS}}^2 &= .9631R^2 - 1.963RT + T^2 + .01847(A^2 + C^2) \\ &\approx (.981R - T)^2 + \frac{A^2 + C^2}{54.1} \end{aligned} \quad (3.12)$$

All derived Eqs. (3.9, 3.10, 3.11, and 3.12) are agree with equations defined in (3.2).

3.2.6. GNSS SIS errors characterization

Our aim is to build the global monitoring of GNSS SIS anomalies. An in-depth characterization of nominal SIS errors is necessary and will help the design of anomaly monitors. In addition, the determination of the distribution of the SIS errors is the main key to build a test statistics and carry out reliability analysis. According to the GPS SIS integrity performance standard [30], for any healthy SIS:

$$\Pr(|I - \text{URE}| > 4.42 * \text{URA-UB}) \leq 10^{-5} \quad (3.13)$$

This standard indicates an underlying assumption that I-UREs are zero-mean Gaussian and properly over-bounded by URA-UB. In addition, one of the key assumptions in RAIM is that large UREs occur on several satellites simultaneously with very low probability [32].

This sections aims at a statistical characterization of nominal GNSS SIS errors with focus on validating these assumptions, and is organized as follow; introduction of SIS error metrics and statistical methods for characterizing GNSS SIS errors. This includes the discussion of long-term SIS accuracy performance and the selection of an appropriate range of data for statistics, presenting the statistics for GNSS. These statistics are used to examine the above assumptions about the distribution of nominal UREs and the correlation among UREs of different satellites. Additionally, two particular issues with GLONASS, ephemeris error growth with propagation distance and geographic dependency, are discussed. Finally, we try to summarize the nominal GPS and GLONASS error performance in terms of the similarities and differences.

3.2.6.1. Statistical Methods

In order to derive the correct distribution for GNSS SIS errors, some facts must be considered before data processing. To start with, the GNSS SIS errors behavior are not Gaussian distributed and includes extreme values, that reside far away from the rest of data bulk, known as outliers. In addition, low sampling rate of observations (5 to 15 minutes) results undetected anomalies that occurs between sampling epochs. In analysis, all observations causing anomalies must be removed/deleted from data before carrying out statistical analysis.

SIS error additional metrics

Two additional metrics are considered, namely I-URE computed for x points spread evenly around the globe, and a global average SIS URE containing only orbital error (GA-URE_O) and reads:

$$\text{GA-URE-orbit} \equiv \begin{cases} \sqrt{(0.98R)^2 + (A^2 + C^2)/49} & \text{for GPS} & (3.14a) \\ \sqrt{(0.98R)^2 + (A^2 + C^2)/61.2} & \text{for Galileo} & (3.14b) \\ \sqrt{(S_1 * R)^2 + (A^2 + C^2)/S_2} & \text{for BeiDou} & (3.14c) \\ \sqrt{(0.98R)^2 + (A^2 + C^2)/45} & \text{for GLONASS} & (3.14d) \end{cases}$$

where R is the radial ephemeris error, A is the along-track ephemeris error, C is the cross-track ephemeris error.

Robust statistics

Since GNSS SIS errors are not Gaussian distributed and are not free from outliers, and our main interest is to reduce the influence of outliers under the estimation process. The traditional statistics such as sample mean and sample standard deviation are affected by extreme samples. Moreover, the sample mean is not necessarily the maximum likelihood (ML) estimator of expected value for non-Gaussian samples. For instance, sample median, rather than sample mean, is the ML estimator of expected value for the samples from a two-sided exponential distribution [47].

Robust mean and variance

To cope with this problem, we use the trimmed mean to measure the central tendency. Analogously, a trimmed standard deviation function is defined by

Eq. (3.15), where α is the significance level and X is our stochastic variable, for instance URE.

$$\text{std}_\alpha = \sqrt{(\text{mean}_\alpha((X - \text{mean}_\alpha(X))^2))} \quad (3.15)$$

In fact, the trimmed mean is a compromise between the sample mean and the sample median, and the trimmed standard deviation is a compromise between the sample standard deviation and the sample median absolute deviation. The value of $\alpha = 0.01$ is free to choose and is preferable to be small i.e., 99% of the core data, to make the result close to the mean or the standard deviation.

Testing for robust normality

Assumptions about normality of observations are commonly used in data analysis by the modeler and help to carry out the inference and hypothesis testing. Most test statistics, for instance the Student t -test and Fisher F -test, rely on normality assumptions. Our aim is to assess the deviation from the Gaussian distribution.

Well-known statistical hypothesis tests of normality, such as the Shapiro-Wilk test [58], Lilliefors test [57], and Jarque-Bera test [56], are so strict that they usually reject the null hypothesis that the SIS error samples comes from a distribution in the normal family. Even worse, common software implementations of these tests can seldom return a meaningful p-value [59] to indicate how far SIS error samples are from normally distributed. Therefore, we use sample kurtosis to quantify normality. Kurtosis (also known as excess kurtosis) is defined as:

$$\gamma = \frac{\mathbb{E}(X - \mathbb{E}(X))^4}{(\mathbb{E}(X - \mathbb{E}(X))^2)^2} - 3 \quad (3.16)$$

where $\mathbb{E}()$ is the expectation operator. As shown in Figure 3.2 a normal distribution has a kurtosis $\gamma = 0$; A sub-Gaussian distribution with light tails usually has a kurtosis $\gamma \leq 0$. A super-Gaussian distribution with heavy tails usually has a kurtosis $\gamma > 0$.

Quantile-Quantile plot

The Q-Q plot allows comparison of two sets, better than the median or the mean comparison. It is often used to test for normality. It is a visualization tool between the theoretical and empirical distributions.

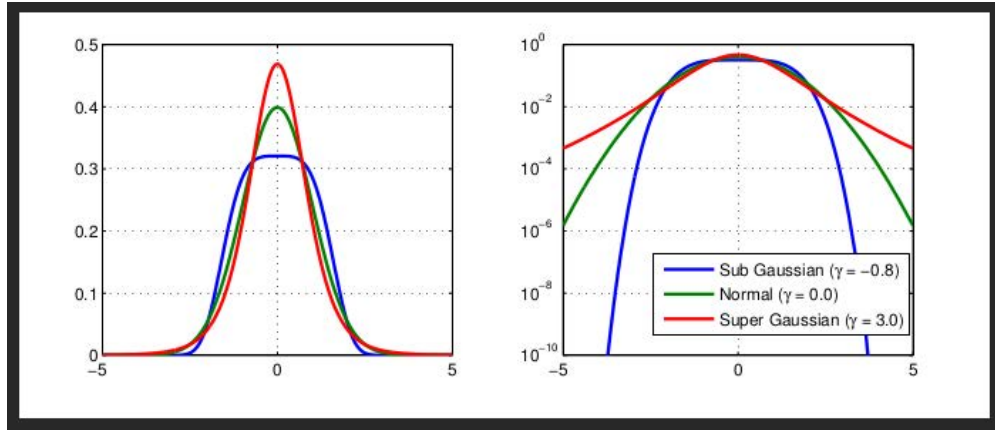


Figure 3.2.: Gaussian distributions (sub-Gaussian, normal and super-Gaussian). Left panel shows the probability distribution function in linear scale, while the right panel shows probability function in logarithmic scale.

3.3. Atmospheric data integrity

The ionospheric behavior closely affects satellite signals, and the variation in the Total Electron Content (TEC) is one of the key indicators of the ionospheric layer. Now, GNSS has become an important tool for monitoring the TEC in real-time. The range error caused by the ionosphere delay in GNSS signals is currently the largest component that affects the accuracy of positioning and navigation determination using GNSS measurements. Modeling the variations of TEC is an effective approach for correcting the ionospheric range error and improving the GNSS accuracy. Measurements from dual-frequency GNSS receiver allow the user in the field to precisely determine the magnitude of the ionosphere delay at their location up to first order. We focus only on ionospheric path delay because is considered as the major threat to navigation and positioning. Other GNSS error sources such as troposphere path delay will be treated as well in this section. Anyway, the aim of this section is to construct some quality indicators that is able to monitor the variation of TEC and the troposphere gradient.

3.3.1. Mitigation of the ionospheric path delay

The ionosphere is part of the Earth's upper atmosphere and it's layer extends from approximately 50 to 1000 km above the surface of the Earth. It consists of gases that have been ionized by solar radiation. The ionization process produces a clouds of free electrons that act as a dispersive medium for GNSS signals (velocity is a function of frequency). It is a pure spatio-temporal process. The spatial and temporal variability has a very significant impact

on GNSS signals traveling from satellites and reaching the user receivers. The dynamics of the ionosphere has remarkable characteristics, the structure and the peak electron density vary strongly with time, geographic location (equator, North and South hemisphere). Moreover, it's variability can be classified as follow:

- Diurnal variation: vary with time of the day.
- Seasonal variation: season of the year.
- Periodical variation: 11 years sunspot cycle.
- Geographical location: Latitude variation.

In addition, the solar radiation strikes the atmosphere with an average power density of $1.37kw/m^2$. This value is known as the solar constant. When solar radiation strikes the atmospheric molecules, these will absorb part of this radiation and a free electron and a positive ion are produced. The ionosphere path delay in GPS pseudo-range observations is the largest error source and can retard radio waves from their velocity in free space by more than 300 ns, on a worst case basis, corresponding to a range error of 100 m. A GNSS signal exhibits code delay and carrier-phase advance when traversing the ionosphere. The important parameter for ionosphere path delay is the total number of electrons (TEC) encountered by the radio waves on its path from satellite to the GNSS receiver. The slant total electron content (TEC) is defined as the electron density integrated along the LOS path. The TEC is expressed in TEC units (TECU), where 1 TECU is equal to 10^{16} electrons in a cylindrical volume with a $1 m^2$ cross section aligned along the LOS.

$$TEC = \int_s^r N_e(s)ds \quad (3.17)$$

The slant TEC (STEC) is calculated along the LOS from a satellite to a receiver and therefore is a function of the satellite and the receiver antenna position. Any way, we are interested in TEC along the local vertical, namely the VTEC. The mapping function that maps the STEC to the VTEC is needed. All users have to correct for this error, and the technique used differs and is based on the user's GNSS receiver.

3.3.1.1. Single Layer Model

The ionosphere is often modeled as a SLM shown in Figure 3.3. The model well known as the thin-shell model. SLM assumes that all free electrons are concentrated in a shell of infinitesimal thickness. The height of this

idealized layer is usually correspond to the expected height of the maximum electron content. Usually, the value of 350 – 450 km are used.

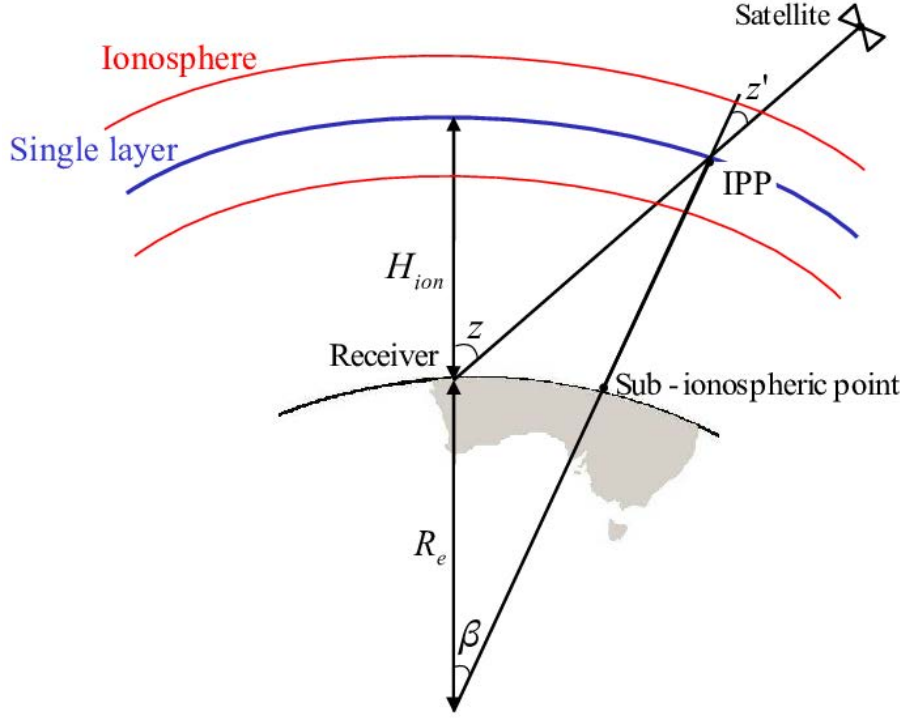


Figure 3.3.: Single Layer Ionosphere Model

The derivation of the latitude ϕ_{ipp} and longitude λ_{ipp} of the IPP is done by the following expression.

$$z = \frac{\pi}{2} - E_l \quad (3.18)$$

$$z' = \arcsin\left\{\frac{R}{R+H} \sin(z)\right\} \quad (3.19)$$

$$\beta = z - z' \quad (3.20)$$

$$\phi_{ipp} = \arcsin\left\{\cos(\beta) \sin(\phi_r) + \sin(\beta) \cos(\phi_r) \cos(A_r^s)\right\} \quad (3.21)$$

$$\lambda_{ipp} = \lambda_r + \arcsin\left\{\frac{\sin(\beta) \sin(A_r^s)}{\cos(\phi_{ipp})}\right\} \quad (3.22)$$

where R is the radius of the Earth. E_l is the elevation angle. H is the height of the ionosphere shell, usually 350 km. The (ϕ_r, λ_r) are longitude and latitude of the receiver. A_r^s is the azimuth angle. The model approximate the shell as a sphere.

1. VTEC value at the IPP and the time t , $\text{VTEC}(t, \phi_{\text{ipp}}, \lambda_{\text{ipp}})$ for L_i , the corresponding delay $I_{r,i}^s$ [m] reads:

$$I_{r,i}^s = \left\{ \frac{1}{\cos(z^t)} \right\} \left\{ \frac{40.3 \times 10^{16}}{f_i} \right\} \text{TEC}(t, \phi_{\text{ipp}}, \lambda_{\text{ipp}}) \quad (3.23)$$

where f_i is the carrier frequency of the signals (Hz).

2. The values of VTEC can be provided in several formats. In case of grid values such as IONEX format, we can interpolate grid point values to any appropriate **IPP** position by a simple interpolation method, for instance the bi-linear interpolation method. The VTEC values are provided at every epoch time intervals in the IONEX file, so the time interpolation should be applied in the sun-fixed coordinates and works as follow:

$$\begin{aligned} \text{TEC}(t, \phi_{\text{ipp}}, \lambda_{\text{ipp}}) = & \left\{ \frac{1}{t_{i+1} - t_i} \right\} \left\{ (t - t_i) \text{TEC}(t_i, \phi_{\text{ipp}}, \lambda_{\text{ipp}}, \omega(t - t_i)) \right. \\ & \left. + (t_{i+1} - t) \text{TEC}(t_{i+1}, \phi_{\text{ipp}}, \lambda_{\text{ipp}}, \omega(t - t_{i+1})) \right\} \end{aligned} \quad (3.24)$$

where $\omega = \frac{2\pi}{86400}$ is the rotation velocity of the sun to the Earth.

3. Modified SLM (MSLM) is just modification of Eq. (3.19).

$$z' = \arcsin \left\{ \frac{R}{R + H} \sin(az) \right\} \quad (3.25)$$

Best approximation of mapping function MSLM is achieved at height of 506.7 Km and scaling factor $a \approx .9782$ with radius of Earth $R = 6371$ Km.

3.3.1.2. Linear combinations of observables

Taking the phase observations L_1 , and L_2 in the unit of meter, the resulting linear combination (LC) defined as

$$L_i = k_{1,i}L_1 + k_{2,i}L_2 \quad (3.26)$$

where $k_{1,i}$ and $k_{2,i}$ are real-valued coefficients defined in Table 3.1.

Due to the dispersive nature of the ionosphere, forming a geometry-free linear combination [Eq. (3.27)] of L_1 and L_2 carrier-phase measurements, the ionospheric delay can be corrected for dual frequency receivers.

$$L_{[GF]} = L_1 - L_2 \quad (3.27)$$

The ionosphere effects in the GNSS signal measurements can be almost completely eliminated by forming the ionosphere-free linear combination (IFLC) [Eq. (3.28)]. IFLC is the one most widely used in GNSS data processing. The advantage of the new derived signal is that the effect of the ionosphere is removed, at least to the first order. The second and third order effect of the ionosphere is at a few millimeter level and still uncorrected. Nevertheless, in recent years models have been developed to account for the higher order ionosphere effects (Bassiri & Hajj, 1993) [99], (Kedar & Hajj, 2003) [100] and (Hoque & Jakowski, 2007) [101]. For the Ionosphere-free linear combination we use the Eq. (3.26) with the following values for $k_{1,i} = f_1^2/(f_1^2 - f_2^2)$ and $k_{2,i} = f_2^2/(f_1^2 - f_2^2)$. The new derived observation equations for code and carrier-phase respectively are:

$$\begin{aligned} L_{[\text{IF}]} &= \frac{1}{(f_1^2 - f_2^2)}(f_1^2 L_1 - f_2^2 L_2) \\ P_{[\text{IF}]} &= \frac{1}{(f_1^2 - f_2^2)}(f_1^2 P_1 - f_2^2 P_2) \end{aligned} \quad (3.28)$$

The drawback of the new signal $L_{[\text{IF}]}$, is that the phase ambiguity N_3 is no longer an integer, which make the process of fixing the ambiguity very difficult and almost impossible to fix. In addition, the noise level is much higher. It should be pointed out that this linear combination does not have true wavelength. However, when in ambiguity fixing using the wide-lane to derive the difference between the number of cycles on L_1 and L_2 in a second step the ionosphere-free linear combination is used to resolve the true number of cycles on L_1 and L_2 . In this step the ionosphere-free linear combination is often referred to as narrow-lane combination. The term “narrow” in this case comes from the fact that the (artificial) wavelength of this linear combination for GPS is only 109 mm.

Table 3.1.: L1–L2 carrier-phase linear combinations.

LC	$k_{1,i}$	$k_{2,i}$	$\alpha_{1,i}$	$\alpha_{2,i}$	λ_i [m]	σ_i/σ_1	I_i
L_1	1	0	1	0	0.190	1	1
L_2	0	1	0	1	0.244	1	1.65
L_{IF}	2.55	−1.55	77	−60	0.006	2.98	0
L_{GF}	1	−1	−60	77	−1	1.41	0.65
L_{WL}	4.53	−3.53	1	−1	0.862	5.74	1.28

3.3.1.3. Ionosphere Broadcast models

For single frequency receivers, the ionosphere broadcast algorithms developed by J. A. Klobuchar [98] can be used to reduce the effect of the ionosphere. This model is used in GPS and BeiDou navigation systems while Galileo uses NeQuick algorithm [31] and for GLONASS no ionosphere correction model is applied. It can correct up to 50% in case of applying the Klobuchar model. The algorithm assumes that all electrons are concentrated in a single layer at 350 km above the Earth surface. The algorithm is simple and take into consideration the number of coefficients used and the computation time.

The α_n and β_n coefficients are uploaded by the master station to the satellites and broadcasted from the satellites to the user in navigation message. In addition, the algorithm uses the elevation angle E_l of satellite and azimuth angle A_z . These parameters must be calculated before calling the algorithm to estimate the ionospheric time-delay.

The Klobuchar broadcast algorithm is very popular and is used in more sophisticated ionospheric monitoring algorithms, for instance in interpolation algorithm Delay Inverse Distance Weighting (IDW) implemented in augmentation system like WAAS, EGNOS, GAGAN systems ([7], [54], [54], [53], [52]) IDW uses Klobuchar as apriori information about IPP and IGP points.

3.3.1.4. Global Ionospheric Map

The Centre for Orbit Determination in Europe (CODE), located at the University of Berne in Switzerland, generates daily global ionospheric maps (GIMs) based on data collected from up to 300 globally distributed GNSS stations and using spherical harmonics for interpolation, Schaer [102]. Maps are grid based ($2.5^\circ \times 5^\circ$) and updated every two hours. Figure 3.4 presents a sample example of a global VTEC map at 12:00 UT on 21th March, 2009 from the CODE Analysis Centre Global Ionospheric Map (GIM).

3.3.2. Ionosphere quality indicators derivation

Three types of indicators will be presented and all have one thing in common, they are related to the computation of the variance of a stochastic variable. These indicators are treated in coming sections. Figure 3.5 shows the steps necessary to compute the indices discussed above. The first step in the processing chain is the data cleaning, which includes detecting and pruning outliers and handling discontinuities caused by cycle-slips and missing observations in the data set. Next, we have to detrend the data set. This means that we construct a time series of the received signal, estimate

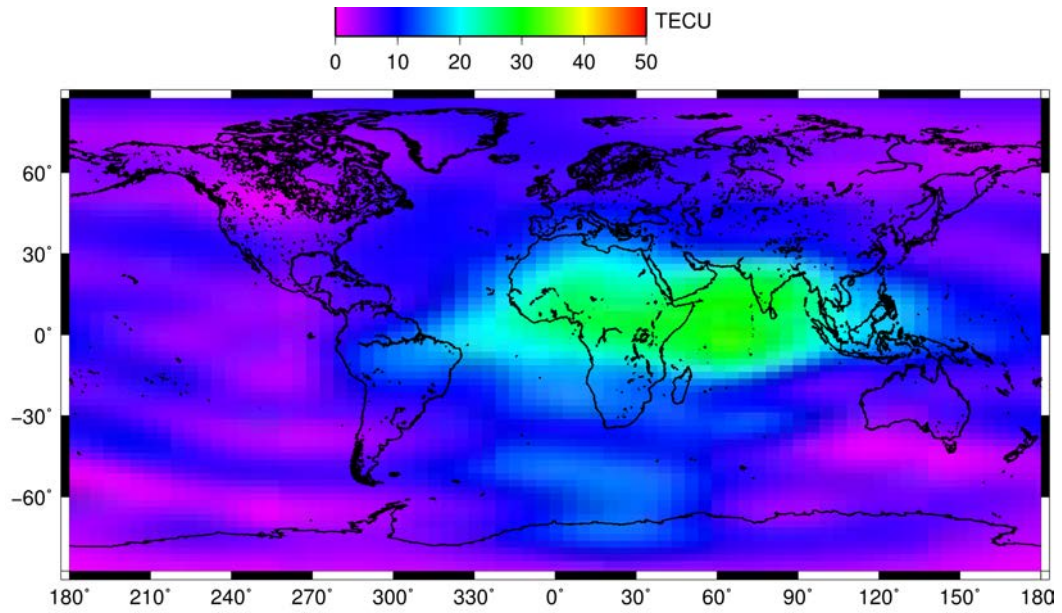


Figure 3.4.: Example of a global VTEC map at 12:00 UT on 21th March, 2009 from the CODE Analysis Centre.
Image credit: Royal Observatory of Belgium (ROB) GNSS Research Group.

the trend, and remove this from the signal. This will remove disturbances due to, for example, the Doppler shift from the relative motion of satellite and receiver. Next, we pass the signal through either a high-pass or low-pass filter, in order to isolate the scintillation components in the signal. Finally, the last step is the computation of the scintillation indices, which is the interesting part. Filtering unwanted signal is necessary in many applications,

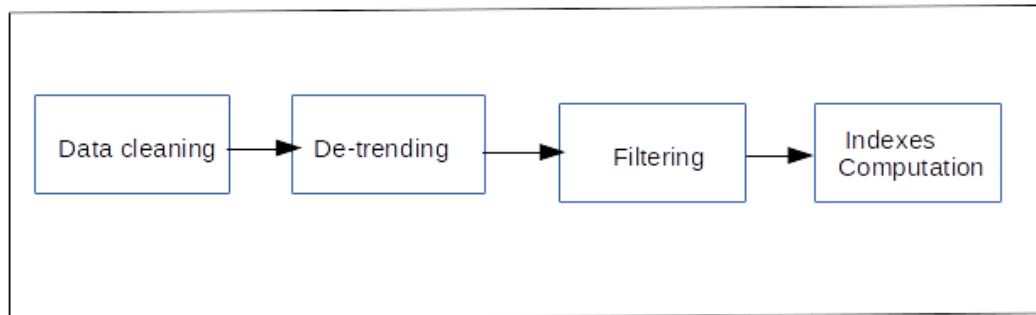


Figure 3.5.: Building blocks of scintillation indices computation.

for instance excluding high frequency components in the computation of S_4 and low frequency components in the computation of σ_ϕ . Figure 3.6 shows low and high filter characterization with three different approximations.

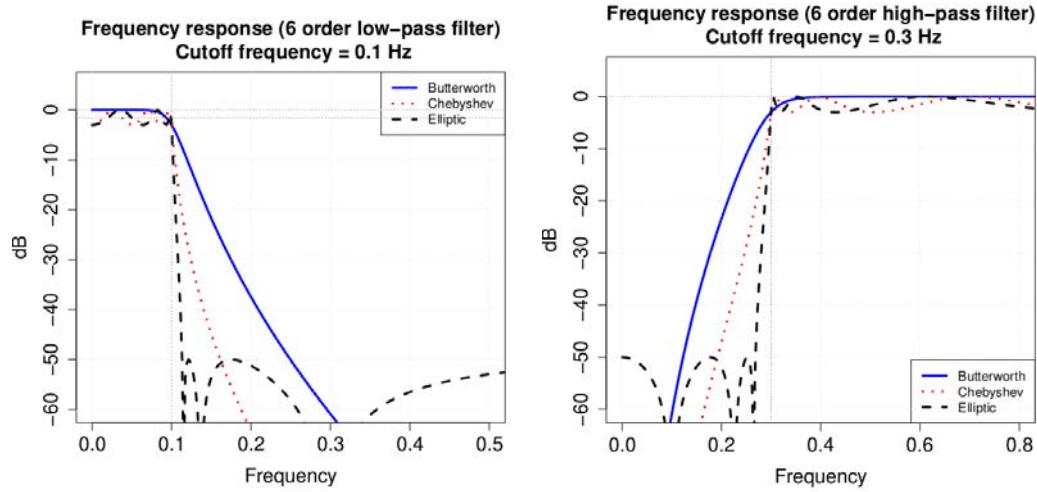


Figure 3.6.: Typical low and high pass filters characteristics used to compute amplitude scintillation index S_4 and phase scintillation σ_ϕ

There are a variety of methods that can be used to design the digital filters to compute S_4 and σ_ϕ . The interested reader is referred to [8]. Other approach most used is the FFT filtering. This procedure transforms the input signal X , then multiplies the coefficients times the transform of X , and finally computes the inverse of FFT. The method is defined by Algorithm 1.

1. Transform the filter coefficients $h(n)$: $H(z) = FFT[h(n)]$, z is complex.
2. Transform the input signal $x(n)$: $X(z) = FFT[x(n)]$.
3. Multiply the complex sequence: $Y(z) = H(z) * X(z)$.
4. Inverse FFT to determine the output: $y(n) = FFT^{-1}[Y(z)]$.
5. Normalization: The IFFT coefficients are the complex conjugates of the Discrete Fast Transform (DFT) coefficients and the output signal shall be scaled down by the length of the signal.

Algorithm 1: Filtering with FFT

3.3.2.1. Amplitude Scintillation Index (S_4)

S_4 is defined as the standard deviation of the signal power normalized to the mean (averaged/expected) value over the interval of interest, usually 1 minute interval. The measured S_4 at L band need to have the effects due to

removal the ambient noise. The ambient noise at L band frequency translates to the relatively high S_4 at lower frequency VHF and UHF frequencies band. The steps needed to compute the S_4 are:

1. Computation of the total amplitude index S_{4_T}

S_{4_T} is defined by the expression:

$$S_{4_T} = \left\{ \frac{\langle SI^2 \rangle - \langle SI \rangle^2}{\langle SI \rangle^2} \right\}^{1/2} \quad (3.29)$$

2. Correction of S_4 by removing the ambient noise

S_{4_T} can have large values due to the ambient noise and must be removed. The removal process is done by estimating the average of the carrier-to-noise ratio (C/N_0) density over the entire evaluation interval and using the estimate to compute S_4 due the ambient noise.

If the signal-to-noise (S/N) density is known, the predicted S_{4_N} due to the ambient noise is given by the expression

$$S_{4_N} = \left\{ \frac{100}{S/N} \left[1 + \frac{500}{19 S/N} \right] \right\}^{1/2} \quad (3.30)$$

Replacing the S/N with the 60 seconds estimate, $\hat{S}/\hat{N}$, we obtain an estimate of the S_4 due to the noise S_{4_N} . Subtracting the square of this value from the square of Equation 3.29 yields the revised value of S_4 .

$$S_4 = \left\{ \frac{\langle SI^2 \rangle - \langle SI \rangle^2}{\langle SI \rangle^2} - \frac{100}{\hat{S}/\hat{N}} \left[1 + \frac{500}{19 \hat{S}/\hat{N}} \right] \right\}^{1/2} \quad (3.31)$$

3. Detrending the signal to achieve stationarity

Detrending the raw signal intensity is accomplished by dividing the original time series by the filtered one, the effect is removable of the low frequency variance. The ratio method is attractive because it's dimensionless and shall not be used if the original data contains negative values and can become a problematic when the fitted line crosses zero.

Detrending by the difference method produces stationarity of the signal under study. The first difference denoted by $\nabla x_t = x_t - x_{t-1}$ eliminates a linear trend, while the second difference can eliminates a quadratic trend, and so on. In this study, both methods are used and

they produce almost the same result, the first difference has a lit higher range than the ratio method.

In order to classify the scintillation activity indicated by S_4 , four categories are defined i.e. $S_4 \leq .25$ is quiet, $S_4 \in [.25, .5]$ is moderate, $S_4 \in [.5, 1]$ is disturbed, and $S_4 > 1$ is severe.

4. Practical considerations

A full understanding of the difference between S/N_0 and the C/N_0 is necessary and useful for the users of the GNSS receivers. The S/N_0 is usually expressed in dB and it refers to the ratio of the signal power and noise power in the given bandwidth while C/N_0 is expressed in dB-Hz, and refers to the ratio of carrier power to noise power per unit bandwidth.

The signal strength indicator measured by the receiver is C/N_0 , the carrier-to-noise density measured in dB-Hz. In the computation of S_4 , the signal-to-noise density S/N_0 is used. The relation between these two quantities is given by the expression:

$$C/N_0 = 10 \log(S/N_0) \implies S/N_0 = 10^{(C/N_0)/10} \quad (3.32)$$

3.3.2.2. Phase scintillation index (σ_ϕ)

Computation of the phase scintillation σ_ϕ is straightforward, the procedures are almost the same as the computation of S_4 . That is, all tasks given by Figure 3.5 are applied, and the difference is the filtering step. The algorithm 1 can be used for σ_ϕ computation, a high-pass filter, usually a Butterworth filter with cut-off frequency $f_c \in [.1, .3]$ Hz. Ouassou et al. [8] show that using local kernel regression method, the optimal smoothing parameter h_{opt} and the cutoff frequency f_c are directly related and the new method eliminates the ambiguities about the determination f_c at different locations on Earth surface.

3.3.2.3. Rate Of Change of TEC Index (ROTI)

The raw material defining the signature of the ionosphere behavior is the TEC. We can derive many products from this raw material, for instance monitoring the variation of this stochastic variable.

Correlation analysis between amplitude scintillations and TEC fluctuations has been explored by a number of authors ([4]; [5]). In these studies, TEC fluctuations are commonly characterized by the parameter ROTI, defined as the standard deviation (taken over $\Delta t \in [1, 5]$ minutes) of the rate of

change of TEC, sampled every $t_s \in [1, 30]$ seconds.

$$\text{ROTI} = \left\{ \frac{\langle \Delta \text{TEC}^2 \rangle - \langle \Delta \text{TEC} \rangle^2}{\langle \Delta t \rangle} \right\}^{1/2} \quad (3.33)$$

where TEC is a vector with components composed of the change in TEC between successive samples, $\langle \Delta \text{TEC}^2 \rangle - \langle \Delta \text{TEC} \rangle^2$ is the variance of the change in TEC between successive samples and Δt is the sampling time.

3.3.2.4. NRTK performance indicators

One inherent weakness of the GNSS Network Real-Time Kinematic (NRTK) software is that the system does not provide the users with a quality indicator for the transmitted signal. This type of information is missing today and routinely requested by the professional users of the GNSS data. This piece of information is referred to as NRTK data integrity. Figure 3.7 shows the high-level functional decomposition of the NRTK data integrity, where the quality control shall be performed at both the system and user levels. Different processing schemes can be used to generate the user corrections: un-, single-, or double-differenced. The user corrections may optionally be further decomposed into dispersive and geometric contributions based on their frequency-dependence. Our aim is to identify the exact locations in the NRTK data processing chain where data quality ought to be inspected and diagnosed. The analysis results should be a list of suspicious satellites that generate anomalous data.

At the network level, a check barrier shall be implemented to check the quality of the user corrections and the uncertainty provided by the covariance matrices. This check guarantees high quality for a simulated reference receiver, known as a virtual reference receiver (VRS) or computation point (CP). This check is referred to as network data integrity. The curved line of the left panel in Fig. 3.7 indicates the output for this computation point. The next check barrier is at the baseline level, where the local data integrity is handled. The raw rover observation data is inspected by the variance weighting algorithm (i.e. the Danish method). The covariance matrix can then be analyzed at the double-difference level to check for stability. The relative positioning between the computation and rover points are handled at this level, as shown in the middle panel of Fig. 3.7. Finally, the last check barrier is the inspection of the rover position accuracy and the construction of the error ellipse.

In order to predict the state of the user in field, some quality indicators related to atmospheric dispersive and non-dispersive residuals are computed by some NRTK software. These performance indicators are IRIU, IRIM,

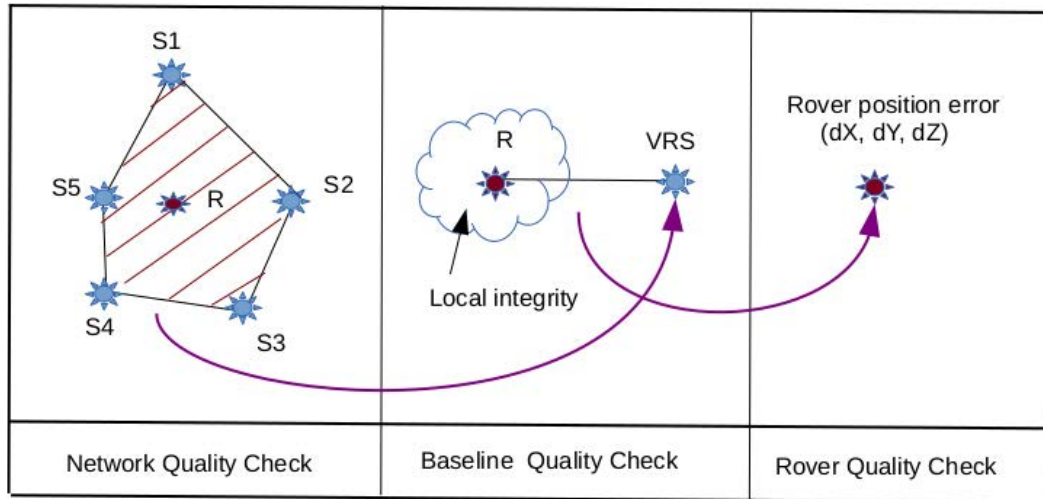


Figure 3.7.: Check barriers of the network RTK data integrity. The left panel shows a network with five reference receivers S_i and a rover R . The middle panel shows the baseline quality check. The right panel shows the rover position error

GRIM and I_{95} and are provided by some NRTK software supplies. These parameters shall be transmitted to the user:

1. Computed TEC.
2. I_{95} Index computation: The I_{95} Index values reflect the intensity of ionospheric activity, i.e., the expected influences onto the relative GNSS positions. The I_{95} values are computed from the ionospheric corrections for all satellites at all network stations for the respective hour. The worst 5% of data are rejected. The highest then remaining value is the I_{95} index value that can be transmitted to the user.
3. IRIM (Predicted Ionospheric Error) values computation.
4. IRIU (Ionospheric Residual Interpolation Uncertainty) computation.
5. GRIM (Predicted Geometric Error) values computation.

IRIU can be used to predict the state of the user in the field under high ionosphere activities, because the linear part of ionospheric residual is normally taken out by the interpolation, the nonlinear part, which is not modeled by the interpolation, will remain in the network correction sent to the rover. In another words, the more linear the ionosphere, the better accuracy of the interpolation and less error “seen” by the rover. Thus, the ionospheric linearity could provide network operator a better idea of the residual errors within the network. Furthermore, it can provide a good estimate of the interpolation error for a possible field user.

One possible ionospheric linearity indicator is Ionospheric Residual Integrity Monitoring (IRIM), which is calculated by omitting one reference station from interpolation and comparing the interpolation results for all satellites at that station with the real measurements, then compute a weighted RMS over all satellites at one epoch, and accumulate the weighted RMS over one hour to get 95% distribution. This can also be considered as integrity monitoring for residual interpolation and ambiguity resolution in the network.

Ouassou et al. [7] shows that an advanced/sophisticated interpolation algorithm will avoid the information loss under interpolating the user biases.

3.4. Troposphere quality indicators derivation

The lower part of the atmosphere, called the troposphere, is electrically neutral and non-dispersive for frequencies as high as about 15 GHz. The raw tropospheric delay in pseudo-range units depends on the vertical delay related to the dry and wet components. About 90% of the tropospheric refraction arise from the dry component and about 10% from the wet component. Due to the temperature, pressure and humidity variations of the atmosphere, the wet component is difficult to model. Besides the path delay caused by the ionosphere, the troposphere path delays are considered as a threat to precise GNSS applications. The aim is to correct the tropospheric delays as accurately as possible by identification of the correct empirical tropospheric model with a mapping function. Wang and Li [103] showed that there is no practical difference between mapping functions with or without meteorological data for wet component derivation. Since the troposphere is height dependent, Feng [103] applied a decorrelation algorithm to achieve improvement of z component.

Several models exist for correction of troposphere path delays. The most popular used in this thesis is the empirical Saastamoinen model [105].

$$\Delta_{trop} = \frac{0.002277}{\cos(z)} \left\{ p + \left(\frac{1255}{T} + 0.05 \right) e - \tan^2(z) \right\} \quad (3.34)$$

where p , T , and e are the total pressure in hPa, absolute temperature of the air in Kelvin, and partial pressure of water vapor in hPa, respectively, derived from the standard atmosphere. h_{rel} is the relative humidity and h is the geodetic height above the MSL.

$$P = 1013.25(1 - 2.2557 \times 10^{-5} \times h)^{5.2568} \quad (3.35)$$

$$T = 15.0 - 6.5 \times 10^{-5} + 2.73.15 \quad (3.36)$$

$$e = 6.108 \times \exp\left\{\frac{17.15T - 4684.0}{T - 38.45}\right\} \times \frac{h_{rel}}{100} \quad (3.37)$$

3.4.1. Tropospheric delays modeling

The GNSS signals take a long time to traverse the air mass, therefore the tropospheric path delay is shortest in the zenith direction than other directions. Due to the spatial and temporal variations of the troposphere, its difficult to define the exact functional relationship caused by this dependency. Often we define the mapping functions to describe the dependence the slant hydro-static (SHD) and the slant wet delays (SWD) into their corresponding zenith delays, ZHD and ZWD, respectively. Note that the mapping functions are always a function of the elevation angle α of the slant signal.

Denote by α the zenith angle, then the computation of SHD and SWD reads:

$$\text{SHD} = \text{ZHD} \times M_h(\alpha) \quad (3.38)$$

$$\text{SHD} = \text{ZWD} \times M_w(\alpha) \quad (3.39)$$

The slant total delay caused by the troposphere is then equal to sum of two quantities, SHD, and ZHD, respectively.

$$\text{STD} = \text{SHD} + \text{ZHD} \quad (3.40)$$

3.4.2. Zenith dry delay

Several mapping functions have been developed in the past, the one most used is the Niell [104] mapping function (NMF). The NMF is accurate, independent of the meteorological data and take into consideration the time of the year.

$$M_N(\alpha) = \frac{1 + \frac{a}{b}}{1 + \frac{1+c}{a}} + h_{[km]} \left\{ \frac{1 + \frac{a_h}{b_h}}{1 + \frac{1+c_h}{a_h}} \right\} \left\{ \frac{\cos(\alpha) + \frac{b}{\cos(\alpha) + c}}{\cos(\alpha) + \frac{b_h}{\cos(\alpha) + c_h}} \right\}$$

The coefficients of the expression 3.41 are provided by tables 3.2 and 3.3, respectively, as a function of station latitude ϕ . If $\phi \in]15, 75[$, the interpolation algorithm must be applied. In addition, the coefficients a , b and c must be corrected for periodic terms by application of the Equation (3.41).

Table 3.2.: Coefficients for Neill's DMF.

ϕ	$\tilde{a} \times 10^3$	$\tilde{b} \times 10^3$	$\tilde{c} \times 10^3$
≤ 15	1.2769934	2.9153695	62.610505
30	1.2683230	2.9152299	62.837393
45	1.2465397	2.9288445	63.721774
60	1.2196049	2.9022565	63.824265
≥ 75	1.2045996	2.9024912	64.258455
ϕ	$a_p \times 10^5$	$b_p \times 10^5$	$c_p \times 10^5$
≤ 15	0	0	0
30	1.2709626	2.1414979	9.0128400
45	2.6523662	3.0160779	4.3497037
60	3.4000452	7.2562722	84.795348
≥ 75	4.1202191	11.723375	170.37206
	$a_h \times 10^3$	$b_h \times 10^3$	$c_h \times 10^3$
	0.0000253	5.49	1.14

$$a(\alpha, DOY) = \tilde{a} - a_p \cos \left\{ 2\pi \frac{DOY - DOY_0}{365.25} \right\} \quad (3.41)$$

where DOY is the day of year and DOY_0 is equal to 28 or 211 for stations located in the Southern and Northern hemisphere, respectively.

3.4.3. Zenith wet delay

One precise way to measure the accuracy of wet delays is by water vapor radiometer. Other methods often used in GNSS is the use of the empirical wet

Table 3.3.: Coefficients for Neill's WMF.

ϕ	$a \times 10^4$	$a \times 10^3$	$c \times 10^2$
15	5.8021897	1.4275268	4.3472961
30	5.6794847	1.5138625	4.6729510
45	5.8118019	1.4572752	4.3908931
60	5.9727542	1.5007428	4.4626982
75	6.1641693	1.7599082	5.4736038

delay correction and the performance depends the accuracy of the zenith wet delay model and the wet mapping function. Due to non-uniform distribution of water vapor horizontally and vertically, capturing the covariance structure of the spatio-temporal process is very complicated, often we end by imposing the stationarity. Anyway, the process noise of the troposphere wet component is modeled as a random walk or first-order Gauss-Markov processes [110, Chap. 2], where the covariance is an exponential function. A Kalman filter is used for this purpose to estimate the wet component as part of the state vector.

3.5. Local data integrity

This section is devoted to investigate the local environment that have direct impact on the user position solution. This includes multipath mitigation, spoofing and jamming.

3.5.1. Multipath mitigation

GNSS signal reaching the receiver from different paths in addition to direct signal is called multipath and is caused by reflections from surrounding objects (near the receiver). As a result, it is highly dependent upon the conditions surrounding the receiver antenna, the type of antenna, and the internal tracking loop algorithms of the receiver. This makes multipath a difficult error to remove, because there is no model that can be used for the general case. The received signal at the receiver is composed of direct and indirect signals, S_d and S_i , respectively.

$$S(t) = S_d(t) + \sum_{i=1}^N S_i(t) \quad (3.42)$$

Indirect signal differs from the direct one in amplitude, phase and in delay (delayed/advanced) depending on surrounding objects reflection characteristics. These differences destroy the shape of the auto-covariance function and make the process to trace the peak very difficult. As a result, reduction of the effect of the multipath can be carried out by any algorithm that is capable to construct all important parts of the auto-covariance function in the tracking loop. Some important characteristics of multipath are:

1. Multipath signal will always arrive after the direct path signal because it must travel a longer propagation path.
2. The multipath signal will normally be weaker than the direct path signal since some signal power will be lost from the reflection. It can be stronger if the direct path signal is hindered in some way.

3. If the delay of the multipath is less than two pseudo-random noise (PRN) code chip lengths, the internally generated receiver signal will partially correlate with it. If the delay is greater than two chips, the correlation power will be negligible
4. Multipath delays are periodic signal with zero average with time correlation period sufficiently large, up to a couple of minutes.

Since the multipath error is not modeled in the GNSS functional model, several methods are proposed to reduce the effect of the multipath, for instance Souza et al. [107] proposed the use of wavelet method to correct for multipath effect. The interested reader on how to reduce the code multipath errors is referred to [108, Chap. 10].

3.5.2. GPS Multipath mitigation algorithm

The most common methods for reducing multipath are improved antenna design (e.g., choke ring ground planes) and careful site selection to model the site specific multipath pattern. This task is achieved by taking advantage of the fixed reflector to antenna geometry at reference stations and of the daily repeatability of multipath. This can be done utilizing techniques such as the Hilbert Huang transformation to decompose the time-shifted post-fit GPS phase signal residuals [109].

Another approach is to include multipath error in the network estimation process as part of the main state vector, which will average out the uncorrelated multipath errors. Unfortunately, it is often not possible to change accuracy or processing time. Therefore, the method of reducing multipath that will be most transparent to the user is to remove it at the signal level within the GNSS receiver itself. That is, including multipath error in the network estimation process as part of the main state vector. In order to mitigate the multipath, we propose a concept based on iterative weighted least square method and only GPS is considered. Our inspiration is taken from PPP method by following the same procedure. Start by the observations equations, estimate/eliminate the nuisance parameters, and finally isolate the parameter of interest, namely the multipath.

3.5.2.1. Derivation of multipath equation

A Kalman filter is used to monitor the multipath at observation domain. The parameters that will be investigated are the signal to noise ratio on L_1 and L_2 , and double difference phase residuals. Starting with basic GPS observables (P_1, P_2, Φ_1 and Φ_2), equations of the code and carrier-phase observations from

satellite k to receiver i in unit of length reads:

$$P_i^k = \rho_i^k + c(\delta t_i - \delta t^k) + c(b_i + b^k) + T_i^k + I_i^k + M_i^k + r_i^k + V_i^k + \epsilon_i^k \quad (3.43)$$

$$\Phi_i^k = \rho_i^k + c(\delta t_i - \delta t^k) + c(a_i + a^k) + T_i^k - I_i^k + m_i^k + r_i^k + v_i^k + \lambda N_i^k + \epsilon_i^k \quad (3.44)$$

Assuming that hardware biases b_i, b^k, a_i and a^k are estimated and removed and the term ρ_i^k also includes relativistic effects r_i^k , phase center offsets and variations (V_i^k and v_i^k) and orbits errors O_i^k . In addition, assume that no cycle slip is present and ambiguities λN_i^k are resolved.

Let $R_i^k = \rho_i^k + c(\delta t_i - \delta t^k)$ and omitting the indices i and k . The simplified observations model reads:

$$P_1 = R + T + I_1 + M_1 + \epsilon_1 \quad (3.45)$$

$$P_2 = R + T + I_2 + M_2 + \epsilon_2 \quad (3.46)$$

$$\Phi_1 = R + T - I_1 + m_1 + \epsilon_1 \quad (3.47)$$

$$\Phi_2 = R + T - I_2 + m_2 + \epsilon_2 \quad (3.48)$$

The ionospheric delay on L_1 and L_2 are related by the expression:

$$I_2 = \gamma I_1 \quad \text{where } \gamma = \left(\frac{f_1}{f_2} \right)^2 \quad (3.49)$$

Performing ionosphere free linear combination on pseudo-range and carrier-phase we obtain

$$P_4 = R + T + \frac{\gamma}{\gamma - 1}(M_1 + \epsilon_1) - \frac{1}{\gamma - 1}(M_2 + \epsilon_2) \quad (3.50)$$

$$\Phi_4 = R + T + N_4 + \frac{\gamma}{\gamma - 1}(m_1 + \epsilon_1) - \frac{1}{\gamma - 1}(m_2 + \epsilon_2) \quad (3.51)$$

N_4 is the phase ambiguity of the iono-free linear combination of carrier-phase. We have ignored the multipath (m_i) and noise (ϵ) delays in carrier-phase compared to the code pseudo-range. Taking the difference between the two Eqs. ((3.50) and (3.51)) we get the approximation:

$$\Phi_4 - P_4 \approx N_4 + \frac{\gamma}{\gamma - 1}(M_1 + \epsilon_1) - \frac{1}{\gamma - 1}(M_2 + \epsilon_2) \quad (3.52)$$

3.5.2.2. Smoothed measurements

Changes in geometric range ρ to the satellite are independent of the technique used code or carrier-phase. Note that the carrier-phase measurement

has better noise properties than the code. Therefore smoothing the pseudo-range by carrier-phase reduces the noise level and improve the overall accuracy. Before smoothing the code pseudo-range with carrier-phase, the measurements shall be cleaned by removing the ionosphere path delay and the hardware biases. The smoothing process is carried out according to the formula:

$$P_{\text{sm}} = \Phi_t - \frac{\left\{ \sum_{i=1}^{n-1} \Phi_{t-i} - P_{t-i} \right\}}{n} \quad (3.53)$$

where P_t and Φ_t are respectively the code and carrier-phase pseudo-ranges at epoch t and are given by Eqs. (3.43) and (3.44). n is the sliding window size and is used as the smoothing buffer. For practical considerations, the smoothing process is initialized in case of carrier-phase discontinuity caused by a cycle-slip. The smoothing parameter n is a configurable and should take values no less than 200 seconds and no more than 1200 seconds. The multipath will not be removed during the initialization step (less than 200 seconds). Combining the Eqs. (3.53) and (3.52) and arranging, we get:

$$P_{\text{sm}} = \Phi_t - N_4 + \frac{\gamma}{\gamma - 1} \underbrace{\frac{\left\{ \sum_{i=1}^{n-1} (M_1 + \epsilon_1) \right\}}{n}}_{(M_1 + \epsilon_1) \text{ mean value}} - \frac{1}{\gamma - 1} \underbrace{\frac{\left\{ \sum_{i=1}^{n-1} (M_2 + \epsilon_2) \right\}}{n}}_{(M_2 + \epsilon_2) \text{ mean value}} \quad (3.54)$$

From Eq. (3.54) we observe that the we are actually computing the mean value of the noise and the multipath on L_1 and L_2 frequencies. Typically, the multipath delays are zero mean on time periods sufficiently large ($N/2$), and at least can be chosen to correspond to the correlation time of the multipath.

3.5.2.3. Multipath monitoring based on Kalman filter

Monitoring the multipath behavior in real time for each observed satellite from a reference receiver is a challenge. The aim is to detect the satellite influenced by multipath and exclude it from the computation or assign a weight to reduce the influence on position estimation. That is, avoid biased solution caused by multipath. For this purpose, the Kalman filter is chosen due to it's recursive characteristic. The user does not need to save previous observations; instead, all filter observations are carried forward in the filter. Kalman filter includes measurement and system models as well as their corresponding stochastic models as illustrated in equations. The noises w_k and v_k of the models are assumed to be independent from each other and distributed according to the normal distribution. Following the notation proposed by Brown and Hwang, the system and the observations equations

are given by [110, Chap. 5] and will not repeated here. The multipath Eq. (3.52) relating the pseudo-range and carrier-phases is supposed to be constant in time and the variation is dominated by the P code multipath and the measurement accuracy.

Multipath state vector definition $\mathbf{x}$

The state vector is defined as $\mathbf{x} = [M^1 \cdots M^x]$, where M^k refers to the multipath of satellite k observed at the reference receiver y , omitted in this case. From the Eq. (3.82), the state vector $\mathbf{x}_k$ depends only on the previous solution $\mathbf{x}_{k-1}$. This is an auto-regressive process of order 1. Since the noise w_k is normal distributed, the process is referred to as the first-order Gauss-Markov process.

Multipath transition matrix definition Φ

From the Eq. (3.52), theoretically, the multipath variation is considered to be constants in time. Since the process under study is a first-order Gauss-Markov process and its auto-covariance function is an exponential. Therefore the choice of $\Phi = \exp\{-T/\tau\}$ for multipath is an appropriate one and is decreasing exponentially. τ and T denote respectively the correlation time and the time difference between epochs. In many applications, the exponential correlation function is often used, because it fits a large number of physical processes with remarkable accuracy. For interested reader is referred to Huang and Brown [110, p. 95], Gelb [111, p. 81] and Leick [112, p. 169].

$$\Phi = \begin{bmatrix} \exp\{-T/\tau\} & 0 & \cdots & 0 \\ 0 & \exp\{-T/\tau\} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \exp\{-T/\tau\} \end{bmatrix} \quad (3.55)$$

or

$$\Phi = [\exp\{-T/\tau\}] \quad (3.56)$$

If $\frac{T}{\tau}$ is constant, then the multipath is modeled as a constant.

Multipath process noise definition Q

The variance of the process noise for correlation time τ is given by Gelb [111, p. 82].

$$Q_{w_k} = E\{ww^T\} = \frac{\tau}{2}(1 - e^{-\frac{2T}{\tau}}) q_k \quad (3.57)$$

where q_k is the variance of the process noise. The initial values of the quantities τ and q_k can be determined empirically by computing the sample mean and the auto-covariance functions.

When τ approaches 0, the process noise is a white noise and no correlation exist between epochs, when τ approaching infinity, the process is a pure random walk.

Updating the state vector

The updating step at epoch $k+1$ is carried out by multiplying previous state vector with transition matrix Φ .

$$\{M_i^j\}_{k+1} = \Phi \{M_i^j\}_k \quad (3.58)$$

The covariance matrix $P_{k+1} = [\text{var}(M_i^j)] = \Phi P_k \Phi^T + Q$, where the variance is given by the expression:

$$\text{var}\{M_i^j\}_{k+1} = \text{var}\{M_i^j\}_k e^{\frac{-2T}{\tau}} + (1 - e^{\frac{-2T}{\tau}}) q_k \quad (3.59)$$

Innovation computation

In order to control the variation of the process under study, the standardized innovations must be computed and tested for variance upper bound. From Eq. (3.52), we see that the computation of innovations requires that the ambiguities are known. Therefore, the right thing to do is to resolve ambiguities with multipath.

3.5.3. Intentional Disturbances of GNSS signals

Disturbances of GNSS signals can be divided into two main classes, the first class known as "*Man-made signals*", and includes jamming and spoofing, while the second class is caused by mother nature. The aim of this section is to present some mitigation algorithms that deal with both problems and provide some practical solutions.

As mentioned before, our modern society depends strongly on the utilization of the GNSS signals. This includes critical government and industry sectors, such as transportation, communication, high learning institutions, health care, defense and finance. At the same time, the availability of low-cost GNSS jamming devices are presenting a serious threat to GNSS and increasing the likelihood of outages to infrastructures relying on GNSS. Another threat to GNSS signals is generating and sending a false signal, that is powerful compared to a weak signal sent by satellite(s).

- I. **Jamming:** Is an intentional interference technique consisting of emitting radio frequency (RF) signals with specific properties and higher power than the target signal, in order to totally or partially block the reception of the latter. Protecting GNSS signals from jamming and spoofing, the reader is directed to [113]. Using the polarization diversity reception which is the same polarization compared to the jamming can perform the jamming monitoring and the analysis [114]. Adaptive Notch Filters have been widely investigated in literature for their low complexity and good performance, and used in many signal processing applications. More recently they have been proven to be an effective solution against jamming interferences, considered more than ever a real threat to Global Navigation Satellite System (GNSS) signals [115].

There is also a type of jamming, called *meaconing*, in which you need only to tune into the real GNSS signals, record them, and then re-transmit them after a certain delay and more power to confuse the receiver. There is no control over the variables, time and position, which it calculates, though there is a certain chance that the opposite will happen, as will be seen in the spoofing section.

It should be noted that the inhibitor device or jammer does not work selectively and has side effects on other systems, such as air traffic control (ATC). They are devices whose use is forbidden unless authorized, but they may be marketed, since there is a great variety of prices and sizes.

Jammers with less than 100 Watts of power are the most dangerous, because they are difficult to detect. A small, cheap 1W inhibitor can cover 20 km².

- II. **Spoofing:** Is a process of sending a false signal in order to disturb the real signal. Spoofing is another intentional interference technique, in which a device transmits a signal analogous to the satellite signal, but which has higher power, in such a way that the GNSS receiver tunes the false signal instead of the real one, and thus calculates an incorrect position or an erroneous time variable. Various algorithms are derived to handle spoofing by using different approaches. Using signal-geometry-based approaches of GNSS spoofing detection [117]. Another approach that is based likelihood ratio test that is based on the pseudorange, which is the measured value of the receiver position, and detects spoofing signals by setting the monitoring interval according to a “*sliding window*” [116]. Using power spectral density to detect the peak of the correlation function [118].

Like a jammer, a spoofer, as the device responsible for impersonating

authentic GNSS signals is known, is illegal to use. There are several techniques to achieve the aim of this cyberattack:

Simulate a GNSS signal. To do this, online databases related to almanacs, ephemeris and PRN codes are used to create the false GNSS signal in simulator software. This technique works if, when emitting the false signal, it is the first one that the target receiver tunes to, after activating its acquisition period. Otherwise, they can be ignored, so sometimes jamming is used as a preliminary step.

Generate an analogue GNSS signal that is synchronized with the real one. It consists of a process of gradual alignment and misalignment between the real and the false signal in terms of time, shape and power; that is, the false information is not imposed directly as in the previous case. To do this, one needs to know the shape of the real signal, hence the spoofer must have reception and emission capabilities.

Nulling. In this technique, the spoofer emits two signals for each real GNSS signal to be supplanted. One of them is the real one but 180° out of phase, which cancels the one emitted by the satellite; the other is the more powerful false signal.

Meaconing. As mentioned in the jamming section, there is a certain likelihood that when meaconing the signal is processed correctly by the receiver, even if it is military-type, and the receiver thus calculates a false position.

The use of GNSS receivers has become widespread, and jamming and spoofing techniques pose an ever-evolving threat, as does technology, resulting in low-cost and small-size equipment with extensive attack capabilities and more sophisticated strategies. Some attacking plans are provided as backup to increase robustness of GNSS, and provide some solutions. For both threats, it is common to increase the power of the real signal using repeaters to make it strong, however, this solution does not suffice. As solutions to jamming the following are proposed:

- Backup systems such as Inertial Navigation System (INS) can be used to aid GNSS.
- Authentication (Galileo Public Regulated Service)
- New receiver algorithms at signal processing level to tackle spoofing and jamming properly.
- Law enforcement. Countries have different policy regarding the use of jamming devices. Some countries allow to buy and use such devices,

some allow to buy but not use the devices like USA, and in some countries it is forbidden to buy or use such devices. The right remedy is enforce the law in terms of money and in worst case jail.

1. **Solutions to tackle jamming:** Are devices that detect a nearby active jammer using signal-processing algorithms, which identify alterations in the received wave, such as frequency peaks.
 - a) *Anti-jamming antennae:* Known as Control Reception Pattern Antennae (CRPA) and they are basically matrices receiving antennae with the ability to modify their reception patterns and thus to reduce them in the direction in which the interference is received, creating a null space to avoid jamming. Therefore, it makes it possible to detect the source direction of the interference and to provide an additional reception gain for the real signals.
 - b) *Inertial devices:* This solution consists of coupling an inertial Measurement Unit (IMU) to a GNSS receiver. Thus, in the event of signal loss, the receiver will make its calculations based on the latest calculated data and those provided by the IMU.
 - c) *Notch filters:* Known as adaptive band-rejection filters to eliminate the region of the frequency spectrum affected by the interference. They must be implemented before the receiver's ADC (Analogue-to-Digital Converter).
2. **Solutions to tackle spoofing:** Some suggestions that require implementation at receiver signal acquisition. This includes:
 - a) *Signal-processing algorithms:* Monitoring and detection of sudden changes in the characteristics of the received signal, such as amplitude, phase or power, or they also check its auto-correlation function. With this technique it is only possible to detect spoofing at the time of the initial attack: after that, it is useless.
 - b) *Radio spectrum monitoring:* Aim is to detect duplicate signals.
 - c) *Matrix of receiving antennae:* it detects the false signal due to its angle of arrival.
 - d) *Encryption:* this solution is used for military services or with authorization and is forbidden for civilian use. The encryption can be symmetric or asymmetric and may be done at the signal level or at the data level. The encryption codes and algorithms used are secret and without them it is impossible

to spoof. Moreover, receivers compatible with this solution must include the classified algorithms, have the relevant key repository and be certified for military use.

At the signal level, only the PRN code is encrypted. This is true of the GPS P (Y) code, which results from encrypting the P code with a W code.

At the data level, only the message is encrypted, not the PRN. The receiver will thus be able to track the signal, but will never know what the message says and, therefore, it will not calculate the falsified position. This is true of Galileo's OS-NMA.

Using a PRN data server: this method makes it possible to verify the transmission from historical PRN data stored without the receiver having them in its internal memory, which offers security to simple receivers.

3.6. Data modeling and algorithms

This section is devoted to investigate the estimation methods used in GNSS to estimate the parameters. Starting with sequential least square and then pass to problems that involve stochastic variables. The studies outcome shall consider comparing the respective advantages, limitations, and numerical efficiency of the various methods. For each method considered, a cost/performance function will presented for validation.

3.6.1. Estimation theory

Systematic errors and uncertainties are common in GNSS observations data processing. Finding optimal estimates for the unknown parameters is absolutely required. Estimators with lower bias and high precision (small variance) are preferable over other estimators. Anyway, one always tries to balance between these two terms in best possible way. Since the bias presents systematic error, it may be reasonable to control the bias before considering the variance reduction of an estimate.

The statistical terms of estimate like consistency, sufficiency, convergence in probability and in distribution are interesting characteristics of any estimates. These estimators are key for building the test statistics in general.

We start by giving some important definitions and basic notations to ease discussions and methods used in this thesis. Let (Ω, Σ, ψ) be a probability space, where Ω is a sample space, Σ is a σ -algebra of subset Ω and ψ is probability measure defined over all members of Σ .

Denote by $\mathbf{z} = (z_1, \dots, z_m)$ the observations, $q(\theta) = q(\mathbf{x}_1, \dots, \mathbf{x}_n)$ parameters vector, $T(\cdot)$ the test statistics, $E(T(\cdot)) = q(\mathbf{x})$ is the expectation, $B(\theta, T(\cdot))$ is the bias and $V = (\theta, T)$ is the variance.

Definition 1. *Unbiased Estimator:* An estimator $T(Z)$ is said to be an unbiased estimator for $q(\theta)$ if $E(T) = q(\theta)$, where E is the expectation operator.

Definition 2. *Minimum variance estimator:* An estimator with minimum variance is often preferred over all other estimators and guarantee the stability of the system. Since our estimator satisfy the unbiasedness (Def. 1), the mean square error(MSE) is equivalent to the variance.

$$E\{[(q(\hat{\theta}) - E(q(\hat{\theta})))^T [q(\hat{\theta}) - E(q(\hat{\theta}))]]\} = \min \quad (3.60)$$

Definition 3. *Consistency:* If $T_n(Z_1, \dots, Z_n)$ is reasonable consistent estimator for $q(\theta)$, if its converges in probability. That is, for all $\epsilon \geq 0$.

$$\lim_{N \rightarrow \infty} \mathbb{P}\{|T_N(Z_1, \dots, Z_N) - q(\theta)| \geq \epsilon\} \xrightarrow{\mathbb{P}} 0 \quad (3.61)$$

Definition 4. *Sufficiency:* We say that $T_n(Z_1, \dots, Z_n)$ is reasonable sufficient statistic for $q(\theta)$, if there exists a determination of the conditional distribution $\mathbb{P}(Z|T(\cdot))$ that is independent of $q(\theta)$.

The information about $q(\theta)$ contained in the sample Z_n is concentrated in the statistics $T(\cdot)$.

3.6.1.1. Least-square adjustment method

Least-square adjustment method is the most used techniques in estimation theory. We consider the problem involving stochastic variables with main focus to advantages of the employed method, limitation, numerical efficiencies and stability. A simple linear model is given by the Eq. (3.62), relating the response vector $\mathbf{z}$ to the variates $\mathbf{H}$. $\mathbf{x} \in \mathbb{R}^{(n \times 1)}$ is the model parameter. The measurement noise $\mathbf{v} \in \mathbb{R}^{(m \times 1)}$ has zero-mean and variance-covariance $\Sigma \in \mathbb{R}^{(m \times m)}$, and $\mathbf{H} \in \mathbb{R}^{(m \times n)}$

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{v} \quad \mathbf{v} \sim \mathcal{N}(\mathbf{0}, \Sigma) \quad (3.62)$$

To obtain an valid solution, the number of measurements must be greater than unknowns ($m > n$). According to the principle of least-square, the opti-

mal estimator $\hat{\mathbf{x}}$ is obtained by minimizing the cost/performance function $C(\mathbf{x})$ of the sum of the squares of the residuals $\mathbf{v}$.

$$C(\mathbf{x}) = \frac{1}{2} \sum_{i=1}^m \mathbf{v}_i^2 = \frac{1}{2} [\mathbf{z} - \mathbf{H}\mathbf{x}]^T [\mathbf{z} - \mathbf{H}\mathbf{x}] \quad (3.63)$$

The residual vector $\mathbf{v}$ is defined as the difference between the true value of $\mathbf{z}$ and the estimated $\hat{\mathbf{z}}$.

Minimization of the cost function, Eq. (3.63), with respect to the vector parameter $\mathbf{x}$ is obtained by setting its gradient with respect to $\hat{\mathbf{x}}$ to zero.

The necessary condition:

$$\nabla_{\hat{\mathbf{x}}} C = \frac{\partial C}{\partial \hat{\mathbf{x}}} = (\mathbf{H}^T \mathbf{H})^{-1} \hat{\mathbf{x}} - \mathbf{H}^T \mathbf{z} = 0 \quad (3.64)$$

The sufficient condition, see computational assumptions Section 3.6.1.2.

$$\nabla_{\hat{\mathbf{x}}}^2 C = \frac{\partial^2 C}{\partial \hat{\mathbf{x}} \partial \hat{\mathbf{x}}^T} = (\mathbf{H}^T \mathbf{H})^{-1} \geq 0 \quad (3.65)$$

where $\nabla_{\hat{\mathbf{x}}} C$ and $\nabla_{\hat{\mathbf{x}}}^2 C$ are respectively the Jacobian and the Hessian. From Eq. (3.64), the normal equation (NEQ) is obtained.

$$(\mathbf{H}^T \mathbf{H})^{-1} \hat{\mathbf{x}} = \mathbf{H}^T \mathbf{z} \quad (3.66)$$

3.6.1.2. Assumptions and distributions

Imposing assumptions in statistical analysis is necessary in order to build test statistics, estimations, computing confidence region and decision making through hypothesis testing. The drawback of assumptions is that they limits the generalization.

Assumptions are always linked to linearity, distribution, independence and variance stability.

- **Linearity assumptions:** Each observation response $\mathbf{z}_i$ can be written as linear function of the i 'th row $\mathbf{H}_i^T$. That is, $\mathbf{z}_i = \mathbf{H}_i^T \mathbf{x}$
- **Computational assumptions:** The term $(\mathbf{H}^T \mathbf{H})^{-1}$ exists.
- **Distribution assumptions:** Test statistics mostly used in hypothesis testing for instance the Fisher F -test, student t -test and others, assume that:
 - $\mathbf{H}$ is measured without error.

- v_i is independent of $\mathbf{z}_i$.
 - v_i is identical with a common defined variance σ^2 .
 - we can write that $v_i \stackrel{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2 I)$.
- **Implicit assumptions:** All observations are equally reliable and should have an equal role in determining the least square results and influencing conclusions.
 - **Additional comments**
 - Vector of stochastic error term $\mathbf{v}$, is not directly observed and often is assumed to follow Gaussian distribution.
 - Vector $\mathbf{e}$ reflects the variance function of the true model error.
 - Taking consideration of GNSS temporal correlation, the LS residuals have different variabilities from one another. This phenomena is called heteroscedastic and is due to dependency of satellite(s) elevation angle E_l . A trade-off between observation quality and the satellite geometry is critical in a GNSS application. Therefore transformations are needed to maintain the temporal correlation properties with homogeneous variance (homoscedastic).

The least square solution of Eq. (3.62) reads:

$$\hat{\mathbf{x}} = (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{z} \quad (3.67)$$

$$\hat{\mathbf{z}} = \mathbf{H}(\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T \mathbf{z} = \mathbf{P} \mathbf{y} \quad (3.68)$$

$$\mathbf{e} = (\mathbf{I} - \mathbf{P}) \mathbf{v} \quad (3.69)$$

where $\mathbf{P}$ is called the prediction (hat) matrix. The ordinary residuals $\mathbf{e}$ depend only on $\mathbf{P}$ and the relationship between $\mathbf{e}$ and $\mathbf{v}$ is established by Eq. (3.69).

Assuming that $\mathbf{v}$ are independent and have a common variance, then Eq. (3.69) indicates that $\mathbf{e}$ are not independent (unless $\mathbf{P}$ is diagonal) and they do not have the same variance (unless diagonal elements of $\mathbf{P}$ are equal).

Residuals $\mathbf{e}$ can be regarded as a reasonable substitute of $\mathbf{v}_i$ if the rows of $\mathbf{H}$ is homogeneous. This means that the diagonal element of prediction matrix $\mathbf{P}$ are approximately equal and off-diagonal elements of $\mathbf{P}$ are sufficiently small. For these reasons, it is preferable to use the transformed version of the ordinary residuals for diagnostic purposes. That it is, instead of using e_i one may use the transformed expression given the function $\mathbf{G}$:

$$\mathbf{G}(\mathbf{e}_i, \sigma_i) = \mathbf{e}_i / \sigma_i \quad (3.70)$$

3.6.1.3. Fitting Parametric Model

Various methods can be considered to fit the parametric model to the sample population. The most used methods in parameter estimations are:

1. Least square parameter estimations (LSPE)

- Ordinary Least Square (OLS): The cost function $C_1(\theta)$ is given by the Eq. (3.63).
- Weighted Least Square (WLS): We choose $\mathbf{x}$ to minimize the cost function $C_2(\mathbf{x})$

$$C_2(\mathbf{x}) = \frac{1}{2} \sum_{i=1}^m \epsilon_i^2 = \frac{1}{2} [\mathbf{z} - \mathbf{H}\mathbf{W}\mathbf{x}]^T [\mathbf{z} - \mathbf{H}\mathbf{W}\mathbf{x}] \quad (3.71)$$

The estimate becomes: $\hat{\mathbf{x}}_W = (\mathbf{H}^T \mathbf{W} \mathbf{X})^{-1} \mathbf{H}^T \mathbf{W} \mathbf{z}$ and $\mathbf{W}$ is a diagonal matrix. The elements of $\mathbf{W}(\mathbf{x})$ present the variances of the measurements.

- Generalized Least Square (GLS): We choose $\mathbf{x}$ to minimize the cost function $C_3(\mathbf{x})$

$$C_3(\mathbf{x}) = \frac{1}{2} \sum_{i=1}^m \epsilon_i^2 = \frac{1}{2} [\mathbf{z} - \mathbf{H}\mathbf{G}\mathbf{x}]^T [\mathbf{z} - \mathbf{H}\mathbf{G}\mathbf{x}] \quad (3.72)$$

The estimate becomes: $\hat{\mathbf{x}}_G = (\mathbf{H}^T \mathbf{G} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{G} \mathbf{z}$. The elements of $\mathbf{G}$ present the variances and the covariances of the measurements.

2. Best Linear Unbiasedness Estimator (BLUE): Our aim is to determine an estimator which is unbiased and has the smallest error variance. The generator of such an estimator is the GMM, see Section (3.6.2). The cost function is the mean square error:

$$C_4(\mathbf{x}) = \mathbb{E}\{(\hat{\mathbf{x}} - \mathbf{x})^T (\hat{\mathbf{x}} - \mathbf{x})\} \quad (3.73)$$

We are seeking an estimator of the form $\hat{\mathbf{x}} = \mathbf{B}\mathbf{z}$ that is linear function of the observations. The solution derivation is given [108, p. 226] where $\mathbf{B} = (\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}) \mathbf{H}^T \mathbf{R}^{-1}$, and $\mathbf{R} = \mathbb{E}(\boldsymbol{\epsilon}^T \boldsymbol{\epsilon})$. $\mathbb{E}(\cdot)$ is the expectation operator.

Replacing $\mathbf{R}^{-1} = \mathbf{G}$ in Eq. (3.72) as the weighting matrix, we get the BLUE estimator. The inverse of the weighting matrix $\mathbf{G}$ is referred to as the precision matrix $\mathbf{P}$.

3. Maximum Likelihood (ML): If our observations come from a Gaussian process, then the procedure is simple by computing the exact likelihood and maximize it with respect to unknown parameter vector $\mathbf{x}$. If

the number of observation is larger, this method suffers to compute the inverse of matrix and determinant of the model covariance matrix. If the observations follow the Gaussian distribution, the MLE and LSE provide the same estimate.

4. Maximum a posteriori Estimator (MAP): Maximization of the posterior distribution, multiplication of the likelihood and the prior distribution.

$$\hat{\mathbf{x}}_{\text{map}} = \operatorname{argmax}\{p(\mathbf{x}_k|\mathbf{z}_k)\} \quad (3.74)$$

5. Minimum Means Square Error (MMSE): Defined as the expectation of the posterior distribution.

$$\hat{\mathbf{x}}_{\text{mmse}} = \mathbb{E}\{p(\mathbf{x}_k|\mathbf{z}_k)\} \quad (3.75)$$

3.6.1.4. Computational difficulties

The term $(\mathbf{H}^T \mathbf{W} \mathbf{H})^{-1}$ must exist ! Note that this solution involves inversion of the expression $(\mathbf{H}^T \mathbf{W} \mathbf{H})^{-1}$, which is a very computationally demanding operation for large systems. In practice, one will therefore choose other solution methods for large normal equation systems, for instance the Choleskey decomposition or Singular Value Decomposition (SVD).

3.6.1.5. Choleskey decomposition (CD)

If the design matrix $\mathbf{H}$ has full rank, then the normal equation system ((3.66)) can be solved effectively using so-called Choleskey decomposition.

3.6.1.6. Singular Value Decomposition (SVD)

What happens if the design matrix $\mathbf{H}$ does not have full rank ?

It is still possible to solve the normal equation system, but it will no longer exist a unique solution. This type of normalization system can normally be solved by so-called singular value decomposition.

3.6.2. Gauss-Markov Model

The Gauss-Markov model (GMM) is given by the Eq. (3.62) with additional assumptions. The expectation $\mathbb{E}(\mathbf{z}) = \mathbf{H}\mathbf{x}$ and the variance/deviance is defined $\mathbf{V}(\mathbf{z}) = \sigma^2 \mathbf{P}$, where $\mathbf{P} = \mathbf{W}$ is the precision/weight matrix and is defined as the inverse of Σ . The matrix $\mathbf{P}$ is assumed to be symmetric and

positive definite (SPD), $\mathbf{H}$ has a full column rank and the number n of observation is always kept greater than the number m of parameters. This model states that the unbiased (Def. 1) least squares estimator $q_1(\boldsymbol{\theta})$ has the smallest mean squared error (Def. 2) of all linear estimators with no bias and is preferable over all other estimator candidates. However, there may exist a biased estimator $q_2(\boldsymbol{\theta})$ with minimum mean square error. Such an estimator sacrifices/trades little bias for larger reduction in variance, such estimators are Lasso and Ridge regressors.

The GMM is very popular and widely used in GNSS data processing. Interested reader is referred to [48, Chap. 3.2]. In addition, the model bears the name of two respected scientists Gauss and Markov. Both have used different approaches and reached the same conclusion. Gauss used the least-square method while Markov used the best unbiased estimator to estimate the parameter vector $q(\boldsymbol{\theta})$. The interested reader is referred to the original papers of Gauss [49] and Markov [50].

3.6.3. Penalized methods

The power of the penalized methods is to improve the variance of an estimator. The reduction of the variance is accomplished by either imposing the constraint on regression coefficients or by regularization of the rate of change of residual errors and local variations. That is, find a best compromise between the goodness-of-fit and the smoothness. The results are the prediction accuracy (low variance) and easy interpretation (lower dimensions). Such biased estimators are commonly used and any method that shrink or sets some coefficients of least squares to zero may results in a biased estimator. The two best-known techniques for shrinking the regression coefficients towards zero are Ridge and Lasso regressors. For a detailed mathematical formulation and discussions, please refer to references [60, Chap. 7.3] and [62, Chap.3].

3.6.3.1. Penalized least squares

In case of the penalized least square (PLS), the cost function defined by Eq. (3.63) becomes [68, p. 5]:

$$C(g) = \sum_{i=1}^m [z_i - g(t_i)]^2 + \alpha \int_a^b \{g''(x)\}^2 \quad (3.76)$$

where $\alpha > 0$ is the smoothing parameter and defines the rate of change between the residuals and local variations. Anyway, minimizing $C(g)$ gives the best compromise between smoothness and goodness-of-fit. A large value

of α will make the penalty term more in action while a small value of α , the first term will be the main contribution.

3.6.3.2. Penalized maximum likelihood

MLE is the most attractive method used for parameters estimation. Like any other methods, MLE sometimes fail to deliver what is expected from the method. An amelioration potential is the penalized ML and profile ML and will not be treated here. The interested reader is referred to Cole et al. [63].

3.6.4. Shrinkage methods

The Gauss-Markov model [48, p. 153] and [73, p. 49] states that for any unbiased estimator, the LS estimates for $\mathbf{x}$ have the smallest variance among all other existing unbiased estimators. Anyway, we can find a biased estimator with a smaller mean square error. That is, sacrificing little bias for a larger reduction of variance.

Bias estimators are used in estimation theory. Shrinkage methods are such techniques that impose constraints on regression coefficients and can be considered as a real competitors to the Gauss-Markov model. The most used are Ridge and Lasso regressors.

3.6.4.1. Ridge regression

Ridge regression shrinks the coefficients by imposing a penalty on their size in L_2 and minimizing a penalized residual sum of squares.

$$\hat{\mathbf{x}}_{\text{ridge}} = \underset{\mathbf{x}}{\operatorname{argmin}} \left\{ \sum_{i=1}^m \left\{ z_i - x_0 - \sum_{j=1}^p h_{ij} x_j \right\}^2 + \lambda \sum_{j=1}^p x_j^2 \right\} \quad (3.77)$$

Equivalently, we can write the Eq. (3.77) differently:

$$\hat{\mathbf{x}}_{\text{ridge}} = \underset{\mathbf{x}}{\operatorname{argmin}} \sum_{i=1}^m \left\{ z_i - x_0 - \sum_{j=1}^p h_{ij} x_j \right\}^2, \quad \text{subject to } \sum_{j=1}^p x_j^2 \leq s \quad (3.78)$$

3.6.4.2. Lasso regression

The Lasso estimate replace the L_2 Ridge penalty $\sum_{j=1}^p x_j^2$ by the L_1 Lasso penalty $\sum_{j=1}^p |x_j|$. The Lasso estimate reads:

$$\hat{\mathbf{x}}_{\text{lasso}} = \underset{\mathbf{x}}{\operatorname{argmin}} \sum_{i=1}^m \left\{ z_i - x_0 - \sum_{j=1}^p h_{ij} x_j \right\}^2, \quad \text{subject to } \sum_{j=1}^p |x_j| \leq k \quad (3.79)$$

3.6.5. Bayesian statistics

The popularity of the Bayesian method is at the peak for the moment. The observations, $\mathbf{z}$ and the unknowns, $\boldsymbol{\theta}$ are treated as stochastic. The Kalman filter belongs to this class. For historical development of this amazing data processing methodology, we refer the interested reader to Blangiardo and Cameletti [61, Chap. 3].

In many applications, it is advantageous to consider parameter vector $\boldsymbol{\theta}$ as observed by a stochastic variable, with apriori distribution $\pi(\boldsymbol{\theta})$. In order to compute the posterior distribution $p(\boldsymbol{\theta}|\mathbf{z})$, which represents the uncertainty about $\boldsymbol{\theta}$. Two components are needed. The likelihood $l(\mathbf{z}|\boldsymbol{\theta})$ and $\pi(\boldsymbol{\theta})$. Applying the Bayesian formula on distributions, we get:

$$p(\boldsymbol{\theta}|\mathbf{z}) = \frac{l(\mathbf{z}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})}{p(\mathbf{z})} \quad (3.80)$$

where $p(\mathbf{z})$ is the marginal distribution of the data and is considered as a normalization constant, so the Equation (3.80) can be written as $p(\boldsymbol{\theta}|\mathbf{z}) \propto l(\mathbf{z}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})$.

3.6.5.1. Prior selection

Based on the nature of parameter of interest $\boldsymbol{\theta}$, two choices are considered to derive the prior distribution $\pi(\boldsymbol{\theta})$, informative and non-informative priors. With informative, the prior information is available from previous experiments or from expert's opinions. Non-informative is to use for instance the uniform distribution, but the most famous is the one proposed by Jeffery [74] and is given by $\pi(\boldsymbol{\theta}) \propto |I(\boldsymbol{\theta})|^{1/2}$ where

$$I(\boldsymbol{\theta}) = -\mathbb{E} \left[\frac{\partial^2 \log f(\mathbf{z}|\boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}'} \middle| \boldsymbol{\theta} \right] \quad (3.81)$$

is the expected Fisher information matrix about the parameter vector $\boldsymbol{\theta}$.

Berger [65] presented a prior based on expected discrepancy measures of information. Another class of generating the prior is the conjugate distributions. The posterior and the prior distributions belong to the same distribution family, often the exponential family.

Anyway, it's impossible to cover all the material for the Bayesian modeling. The reader is referred to the books by Berger [66, Chaps. 4–7], and Rowe [67].

3.6.5.2. Kalman-Filter

Kalman Filter (KF) is a weighted recursive least square algorithm, satisfying the following three general requirements of optimality conditions;

- Unbiasedness (Definition 1).
- Consistency (Definition 3).
- Minimum variance (Definition 2).

KF focuses attention on the state vector of the system $\mathbf{x}$, because it contains all necessary and relevant informations needed to investigate the system. This class of estimation is referred to as Bayesian state estimation. One try to estimate the posterior distribution based on all available information provided by the system, including current measurements and the historical data known as prior info.

KF equation definitions

Popularity of KF is due to its recursive characteristic, the user does not need to save previous observations; instead, all filter observations are carried forward in the filter.

In order to make inference about the dynamic system, at least two main equations are needed; the measurement and system models. In addition, their corresponding stochastic models as illustrated in equations. One of the characteristics of the noises, namely, w_k and v_k , of the two models are mutually independent and zero-mean normal distributed with variance-covariance $\mathbf{Q}$ and $\mathbf{R}$.

Following the notation given by Brown and Hwang [110, Chap. 5], the system and the measurement equations are given respectively by Eqs.(3.82) and (3.83).

State-space system model:

$$\mathbf{x}_k = \Phi_k \mathbf{x}_{k-1} + \mathbf{w}_k \quad \mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_k) \quad (3.82)$$

State-space measurement model:

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k \quad \mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_k) \quad (3.83)$$

where

- $\mathbf{x}_k$ $(n \times 1)$ state vector at epoch k ,
- Φ_k transition matrix from epoch $k - 1$ to k ,
- $\mathbf{x}_{k-1}$ state vector at epoch $k - 1$
- w_k system noise at epoch k
- $\mathbf{Q}_k$ covariance matrix of system noise w_k
- $\mathbf{z}_k$ $(m \times 1)$ measurements vector at epoch k ,
- $\mathbf{H}_k$ $(m \times n)$ design matrix that relate the state vector to
 measurements at epoch k
- $\mathbf{v}_k$ $(m \times 1)$ measurement noise at epoch k and assumed to
 white noise
- $\mathbf{R}_k$ $(m \times m)$ covariance matrix of measurement noise $\mathbf{v}_k$

Eq. 3.82 is known as vector auto-regressive process of order 1 (VAR(1)) and describes the evolution of the state $\mathbf{x}_k$ with time. Generally speaking, one way to introduce the dependency between $\mathbf{x}_k$ and $\mathbf{x}_{k-1}$ is through an auto-regressive process. For detailed information of VAR model, the reader is referred to [70, p.302]. The uncertainties are expressed by the error covariance matrix $\mathbf{Q}$. The second equation (3.83) relating the state vector $\mathbf{x}_k$ and the measurement $\mathbf{z}_k$ is modeled as GMM Section 3.6.2. However, the meaning of the Kalman filter equation pairs is given. We can replace any model of these two equations by a competitive one that can improve the results and provide filter stability. The procedures of how to alternate between estimations, corrections and innovations computation are given by algorithm 2.

The computation steps needed for the filter are:

Optimal choice of the Kalman filter gain

The key of the KF is the optimal choice of the Kalman filter gain $\mathbf{K}_k$. $\mathbf{K}_k$ is chosen to minimize the length of the estimation error vector. This is equivalent to minimize the trace of the error covariance matrix (diagonal elements), corresponding to the total variance of our estimator. It is clear that the relationship between these two quantities is:

$$\mathbb{E}\{(\hat{\theta} - \mathbb{E}(\hat{\theta}))^T (\hat{\theta} - \mathbb{E}(\hat{\theta}))\} = \text{trace} \quad \mathbb{E}\{(\hat{\theta} - \mathbb{E}(\hat{\theta}))(\hat{\theta} - \mathbb{E}(\hat{\theta}))^T\} \quad (3.84)$$

The Kalman filter $\mathbf{K}_k$ gain is a very important parameter to monitor in order to guarantee the stability of the filter. The $\mathbf{K}_k$ is directly involved in

Algorithm 2: Conventional Kalman filter main processing loop

```
1 Step 1: Enter prior estimate of the state vector  $\hat{\mathbf{x}}_0^-$  and its  
   corresponding error covariance  $\hat{\mathbf{P}}_0^-$ .  
2 for  $k = 1, 2, \dots$  do  
3   Step 2: Compute the Kalman gain  $\mathbf{K}_k = \hat{\mathbf{P}}_k^- \mathbf{H}_k (\mathbf{H}_k \hat{\mathbf{P}}_k^- \mathbf{H}_k^T + \mathbf{R}_k)^{-1}$   
4   Step 3: Update the estimate  $\hat{\mathbf{x}}_k^+$  with new information  $\mathbf{z}_k$ .  
    $\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k (\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_k^-)$   
5   Step 4: Compute the error covariance  $\hat{\mathbf{P}}_k^+ = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \hat{\mathbf{P}}_k^-$   
6   Step 5: Project a head:  $\hat{\mathbf{x}}_{k+1}^- = \Phi \hat{\mathbf{x}}_k^+$  and  $\hat{\mathbf{P}}_{k+1}^- = \Phi \hat{\mathbf{P}}_k^+ \Phi^T + \mathbf{Q}_k$   
7 end
```

the updating process of the state vector $\hat{\mathbf{x}}_k^+$ and the error covariance matrix $\hat{\mathbf{P}}_k^+$ as shown in steps 3–4 in algorithm 2. This is easily done by inspection of diagonal elements of $\mathbf{K}_k$. Total and specific variances are other options.

3.6.5.3. Heywood Case

The output from Kalman filter(s) need to be investigated based on epoch basis. Iteration between estimation and updating steps causes some element of the main diagonal of the error covariance matrix to become negative. The main diagonal elements correspond to the variance. For statisticians, this phenomena is referred to as the Heywood case [64].

In social science studies where the correlation analysis is the main issues, the most common methods used are for instance the factor analysis (FA) and the structural equation modeling (SEM). The result from different analysis shows the existence of improper solutions and the Heywood case. That is, the solution can not be obtained due to the correlation and the variance estimates are respectively greater than one and less than zero. A statistical effort devoted to understand when these events are most likely to occur and the identification of the main source of the problem. Among them are; outliers [76, Bollen 1987], non-convergence and under-identification [Van Driel 1978] [80], empirical under-identification [77, Rindskopf 1984], and structurally misspecified models [78, Chen 2001].

In the field of navigation and surveillance, similar problems have occurred and been examined while the employment of Kalman filter and the most appropriate reasons are summarized as follow; process noise uncertainties, transition matrix is ill-conditioned, this means that a small change in the constant coefficient matrix results in a large change in the solution, effect of incorrect a priori statistics (the need for initialization of ill-conditioned caused by large a priori, we often use scaling), scaling some variables and

leaving others cause the filter to produce the negative variance, it's better to be avoided. Scaling is a dangerous task and can produce greater errors and unreliable Kalman gain.

Improvement suggested by experienced modelers are; Gelb [111, p. 286], Bierman [81, pp. 68-112], Maybeck [82, p. 368], Grewal et al [83] and others can be summarized as follow:

- State-noise compensation.
- Joseph's stabilized Kalman filter.
- Conventional Kalman filter with lower bound.
- Potter-Schmidt square root filter.
- U-D factorization, the modified weighted Gram Schmidt (MWGS) time propagation.
- Fading memory filter.
- Sequential root square filter (SRIF).

New suggestions to avoid the Heywood case

Another possible way to avoid the Heywood case of the conventional Kalman filter is by model substitutions. That is, replacing the Eqs. (3.83) and (3.82) by a competitive models.

- Measurement model, Eq. (3.83): The GMM can be replaced with any penalized methods defined in Section 3.6.3.
- System model, Eq. (3.82): The VAR(1) can be replaced with any higher order VAR(.) process, for instance VAR(2), an $ARMA(p, q)$ process or Holt-Winter algorithm [71, pp. 322-326].

Anyway, the key in GNSS data processing is to provide a continuous, stable and correct error covariance matrix (EVCN) on epoch by epoch basis.

3.6.6. Reliability analysis

Reliability refers to the controllability of observations, i.e., the ability to detect and remove the blunders and to estimate the effects of undetectable errors may have on a solution [19, Chap. 2].

The quality checks to assess the correctness of the model is achieved through reliability and precision analysis. Here, reliability refers to the trustworthiness of the observations. This is assessed using redundant observations

to detect and identify any model errors (internal reliability), while also expressing the influence of undetected model errors (external reliability). Precision, on the other hand, is characterized by the variance in the observations. In any case, the procedure is straight-forward: we construct test statistics from the model residuals, and use rigorous hypothesis testing to detect and estimate problems with our model.

The elementary building blocks of the GNSS reliability monitoring are: minimal detectable bias (MDB), minimal detectable effect (MDE), and bias to noise ratio (BNR). For deeper insight on this subject, the reader is referred to Leick [112, Chp. 4.10], Kuusniemi et al. [17], Baarda [14] and de Jong et al. [85, Section 3.2].

Anyway, the key in reliability analysis are the improved stochastic model by imposing the variance function based on elevation angle (E_l) and check for blunders based on Detection Identification and Adaptation (DIA) principle [16].

3.6.6.1. Detection and Identification of gross errors

Methods have been developed that can detect and identify gross errors in the observation material before making any further analysis. Such methods are:

1. **Data Snooping:** Is the process of eliminating the observation, which produces the largest normalized residuals. That it, separates the inliers from outliers using the estimated normalized residuals.
2. **τ -test:** The t-distribution is defined as the quotient between the standard normal distribution ($N(0,1)$) and the squared root of Chi-square divided by m, where m is the degree of freedom.

$$t = \frac{N(0,1)}{\sqrt{\chi_m^2/m}} \quad (3.85)$$

The τ -test make use of the a posterior variance of unit weight ($\hat{\sigma}_{v_i}$) as estimated from observations and reads:

$$\tau_i = \frac{v_i}{\hat{\sigma}_{v_i}} = \frac{N(0,1)}{\sqrt{\chi^2}} \quad (3.86)$$

3. **Multiple τ -test:** Works best for iterative solutions by repetition of τ -test until no blunders are present in the data.

Some important remarks to consider:

- **Error types:** Statistical hypothesis testing is a process of conducting and reporting the outcome of an experiment with diagnostic evaluation and decision making. Two types of errors can occur. The first is a *Type I error*, that we reject H_0 when it is true. The second is a *Type II error*, we fail to detect that H_0 is false. Figure 3.4 summarizes these errors.

	H_0 true	H_0 false
Accept H_0	None	Type II
Reject H_0	Type I	None

Table 3.4.: Error types in statistical hypothesis testing. The diagonal entries, i.e. accepting a true hypothesis or rejecting a false one, are not errors at all. The off-diagonal elements, i.e. rejecting a true hypothesis or accepting a false one, are referred to as Type I and II errors, respectively.

Results of any test statistics depend strongly on two pieces of information:

1. whether the null-hypothesis H_0 was rejected or accepted.
2. the level of significance α at which the test is carried out.

$$\begin{aligned}\alpha &= P(\text{Type I error}) & \beta &= P(\text{Type II error}) \\ &= P(\text{Reject } H_0 | H_0 \text{ true}) & &= P(\text{Accept } H_0 | H_0 \text{ false})\end{aligned}$$

- **Chi-square test:** The test is used often to carry out the variance comparison and computing the confidence interval for σ^2 . That is, we need to investigate the properties of sums of the form $\sum_{i=1}^n v_i^2$, where $v_i \sim N(0, 1)$.
- **Signature of WLS:** Is the reduction of sums of squared residuals $\sum_{i=1}^n \left(\frac{z_i}{w_i}\right)^2$.
- **WLS components:** Writing components for WLS ($\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{v}$)

$$\hat{\mathbf{x}} = (\mathbf{H}^T \mathbf{W} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{W} \mathbf{z} \quad (3.87)$$

$$\mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}} = (\mathbf{H}^T \mathbf{W} \mathbf{H})^{-1} \quad (3.88)$$

$$\hat{\mathbf{z}} = \mathbf{H}(\mathbf{H}^T \mathbf{W} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{W} \mathbf{z} = \mathbf{P} \mathbf{z} \quad (3.89)$$

$$\mathbf{Q}_{\hat{\mathbf{z}}\hat{\mathbf{z}}} = \mathbf{H} \mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}} \mathbf{H}^T \quad (3.90)$$

$$\hat{\mathbf{v}} = (\mathbf{I} - \mathbf{P}) \hat{\mathbf{z}} \quad (3.91)$$

$$\mathbf{Q}_{\hat{\mathbf{v}}\hat{\mathbf{v}}} = \mathbf{Q}_{\mathbf{z}\mathbf{z}} - \mathbf{Q}_{\hat{\mathbf{z}}\hat{\mathbf{z}}} \quad (3.92)$$

3.6.6.2. Estimation of gross errors

Rough errors usually estimate along with the unknowns by adding a new column to the design matrix with a value of one for observation as should be tested for coarse errors and zero otherwise.

$$\hat{x}_i = (H^T \quad v_i) \longrightarrow H_1 = \left(\begin{array}{c|c} & 1 \\ H & \vdots \\ & 0 \end{array} \right) \quad (3.93)$$

Estimation of the blunder ∇_i together with unknown parameters reads:

$$\hat{x}_i = (H_i^T W H_i)^{-1} H_i^T W z \longrightarrow \hat{x}_i = (\dots \nabla_i) \quad (3.94)$$

The associated co-factor reads:

$$Q_{\hat{x}\hat{x}} = (H_i^T W H_i)^{-1} = \left(\begin{array}{c|c} - & - \\ - & \vdots \\ - & q_{\nabla\nabla} \end{array} \right) \quad (3.95)$$

A posterior standard deviation ($\hat{\sigma}_0$) without impact of blunders reads:

$$\hat{\sigma}_0 = \frac{1}{n-p-1} (v_i^t W v_i) \longrightarrow v_i = H_i \hat{x}_i - z \quad (3.96)$$

The associated standard deviation of ∇_i reads:

$$\hat{\sigma}_{\nabla} = \hat{\sigma}_0 \sqrt{q_{\nabla\nabla}} \quad (3.97)$$

3.6.6.3. Inspection of gross errors

The impact of gross errors have on the solution computation depends strongly on the user-satellites geometry and observation weights. Our aim is to investigate if our observations contain blunders. First, we have to define the test statistics to carry out the hypothesis testing. The test statistics candidate $T = \frac{\hat{\nabla}_i}{\hat{\sigma}_{\nabla_i}} \sim T_{n-p-1}$ is chosen for this purpose.

$$\begin{aligned} H_0 &: \quad \nabla_i = 0, \\ H_1 &: \quad \nabla_i \neq 0, \end{aligned} \quad (3.98)$$

Under H_0 , that no gross error is present in observation (i), while the alternative hypothesis H_1 , means that there is a influence of blunders. Choosing significance level of the test ($\alpha = 95\%$ or 99%). If the test statistics T exceeds a threshold value (T_α), the source of possible error is detected. That is, if $|T| > T_{(n-p-1, 1-\alpha/2)}$, we reject the null hypothesis H_0 . For adaptation, one way to adapt would be to simply discard the observation with larger value and repeat the test until no blunders are present in the data.

3.6.6.4. Internal reliability

Internal reliability describes the ability of redundant observation to detect and identify specific model errors. By an observation redundancy we mean the proportion of gross errors that are shown in the observation residual errors. The redundancy matrix $\mathbf{R}$ is defined as follow:

$$\mathbf{R} = \mathbf{Q}_{\hat{v}\hat{v}}\mathbf{P} \quad (3.99)$$

where $\mathbf{P} = \mathbf{W} = \Sigma^{-1}$. $\mathbf{P}$ is the precision matrix, $\mathbf{W}$ is the weight matrix, Σ is the variance-covariance matrix. The residuals variance-covariance matrix $\mathbf{Q}_{\hat{v}\hat{v}}$ reads:

$$\mathbf{Q}_{\hat{v}\hat{v}} = \mathbf{P}^{-1} - \mathbf{H}\mathbf{Q}_{\hat{x}\hat{x}}\mathbf{H}^T \quad (3.100)$$

The redundancy of observation (i) is the diagonal element r_{ii} of $\mathbf{R}$. Some facts to consider:

1. $\sum_{i=1}^n r_{ii} = n - R(\mathbf{H}) = df$ (degree of freedom), R is the rank of the design matrix, and n is the number of parameters.
2. $r_{ii} = p_i q_i$, where q_i is the diagonal element of co-factor $\mathbf{Q}_{\hat{v}\hat{v}}$, and p_i is the weight of the i 'th observation.
3. The remaining gross errors are estimated using the endpoints of a confidence interval around the estimated gross errors and reads:

$$\nabla_{\max_i} = \max \left\{ \left\| \hat{\nabla}_i \pm \hat{\sigma}_{\nabla_i} T_{(n-p-1, 1-\frac{\alpha}{2})} \right\| \right\} \quad (3.101)$$

3.6.6.5. External reliability

External reliability expresses the influence of undetectable model errors on the parameters of interest, i.e., coordinates. That is, we try to answer the question: what the effects of undetectable blunders have on the parameters? Our aim is to minimize the impact of blunders on the parameters.

$$\nabla_x = (\mathbf{H}\mathbf{P}\mathbf{H})^{-1}\mathbf{H}\mathbf{P}\nabla_{\max} \quad (3.102)$$

where $\nabla_{\max}^T = (\nabla_{\max_i}, 0 \dots 0)$ is the largest remaining blunder in observation (i).

3.6.6.6. Error ellipse

With error ellipse, our main objective is to display the user position error in the horizontal plan (2D) and their corresponding precision. Let the variance-covariance matrix Σ of the horizontal position (x, y) be given by

$$\Sigma = \begin{pmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{yx} & \sigma_y^2 \end{pmatrix} \quad (3.103)$$

Semi-major axis a , the semi-minor axis b , and the orientation angle λ of the standard ellipse follow from:

$$a^2 = \lambda_{\max}^2 = \frac{1}{2}(\sigma_x^2 + \sigma_y^2) + \sqrt{\frac{1}{4}(\sigma_x^2 + \sigma_y^2)^2 + \sigma_{xy}^2} \quad (3.104)$$

$$b^2 = \lambda_{\min}^2 = \frac{1}{2}(\sigma_x^2 + \sigma_y^2) - \sqrt{\frac{1}{4}(\sigma_x^2 + \sigma_y^2)^2 + \sigma_{xy}^2} \quad (3.105)$$

$$\tan 2\lambda = \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2} \quad (3.106)$$

where $\lambda_{\max}$ and $\lambda_{\min}$ are the largest and the smallest eigenvalues of Σ

Remarks

Practical consideration requires the use of a single scalar quantities as judging number that say something about precision. We list some of them:

1. The distance Root Mean Squared (DRMS), which expresses 2D precision

$$\text{DRMS} = \sqrt{\sigma_x^2 + \sigma_y^2} \quad (3.107)$$

This expression is valid for the range $.2 \leq \frac{\sigma_x}{\sigma_y} \leq 1$, assumed that $\sigma_x \leq \sigma_y$.

2. Mean Radial Square Error (MRSE), which expresses 3D precision

$$\text{DRMS} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} \quad (3.108)$$

3. Circular Error Probable (CEP)

$$\text{CEP} \approx .589(\sigma_x + \sigma_y) \quad (3.109)$$

4. Spherical Error Probable (SEP) known as 50% probability sphere

$$\text{CEP} \approx .51(\sigma_x + \sigma_y + \sigma_z) \quad (3.110)$$

Another quality indicators describing the influence of the user-satellite geometry on the precision of the parameters to be estimated are Dilution Of Precision (DOP) values. The diagonal elements of the expression given by Eq. (3.88) are used, since they describe the variances of the unknown parameters. Consider the parameter vector ($\theta = (x, y, z, c\delta t)$), then the geometry quality indicators read:

1. Geometric DOP (GDOP)

$$\text{GDOP} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 + \sigma_{c\delta t}^2} \quad (3.111)$$

2. Position DOP (PDOP)

$$\text{PDOP} = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2} \quad (3.112)$$

3. Horizontal DOP (HDOP)

$$\text{HDOP} = \sqrt{\sigma_x^2 + \sigma_y^2} \quad (3.113)$$

4. Time DOP (TDOP)

$$\text{TDOP} = \sigma_{c\delta t} \quad (3.114)$$

For more information about the derivation of the quality indicators presented in this section, the interested reader is referred to [29, 85, 112].

3.6.7. Receiver Autonomous Integrity Monitoring

3.6.7.1. GNSS augmentation systems Integrity

GNSS augmentation systems were developed specifically for applications where safety of life is an absolute priority, for instance the aviation, maritime and land users. These systems include SBAS (WAAS, GAGAN, EGNOS and others) and GBAS. Failures and system malfunctions must be detected, repaired if possible and reported to the users. The information provided by such systems can be grouped into three main components:

1. extra ranging sources using GEO communication satellites;
2. a vector of corrections to the GPS signal-in-space, including the ionospheric path delay, clock and ephemeris corrections;
3. integrity monitoring function to alert the users of out of tolerance operations.

Since the ionosphere path delay is a major threat to navigation in nominal operation, the augmentation system processing facilities calculate the error bounds for ionospheric corrections, called Grid Ionospheric Vertical Error (GIVE), and a combined error bound for the clock and the ephemeris corrections called the User Differential Range Errors (UDRE). The correction vector is broadcast to the users via geostationary satellites and used to compute the confidence bounds of the position errors VPL and HPL, respectively. Anyway, the user needs to know the average error level and the corresponding uncertainty of parameters that have direct influence on position solution degradation. Interesting literature on how to estimate the ionospheric error bounds for WAAS [86, 87], guaranteeing the integrity to all users in the coverage area [88], general information on WAAS including message types [89] and detailed algorithms on WAAS performance standards [75].

Test statistics for exceeding events determining the VPL and HPL are based on the Markov or Chebyshev inequalities [90, p. 77]. These inequalities enable us to compute the error bounds on probabilities when only the average error level (the mean error function) and the associated uncertainty (the variance function) measured in terms of standard deviation are known. The probability distribution is assumed to follow the Gaussian distribution with a finite variance σ^2 .

Let us use the notation $\mathbf{p}_e$ for the computed position error, by α the integrity risk value and by κ the protection levels VPL or HPL, respectively. Then the integrity equation is given by Eq. (3.115).

$$P[\|\mathbf{p}_e\| \geq \kappa] \leq \alpha \quad (3.115)$$

A more powerful version of the above is given by the Borell-Tsirelsen-Ibraginov-Sudakov inequality, which gives a universal bound for the excursion probability. For interesting discussions on the computation of the excursion probabilities of Gaussian processes, the reader is referred to [79, Chap. 2].

3.6.7.2. GNSS receiver-based integrity solution

The next type of integrity solution is the receiver-based measurement integrity. The algorithms developed for this purpose are receiver autonomous integrity monitoring (RAIM) family and fault detection and exclusion (FDE) algorithms. For more details about these methods, we refer the reader to e.g. Kaplan and Hegarty [84, pp. 346-360], Prasad and Ruggieri [96, pp. 62-65], and Grewal et al. [97, pp. 106-201]. These algorithms are developed to support primarily aviation applications that are safety-critical in nature. These approaches simply detect and exclude unhealthy satellites from the

computations; in other words, the receiver ensures good measurements at the cost of some information. However, when multiple unhealthy satellites are observed, they are statistically likely to cancel each others effects. This means that the receivers throw away information that can be quite valuable when processed using the right statistical techniques. Due to this fact, the integrity drawn from augmentation systems is typically considered more available than receiver-based integrity [84, Section 7.5]. The inputs to the RAIM algorithm are the standard deviation of the measurement noise, the measurement geometry, and the maximum allowable probabilities for a false alert and a missed detection. The output of the algorithm is the horizontal protection level (HPL), which is the radius of a circle centered at the true aircraft position that is assured to contain the indicated horizontal position with the given probability of false alert and missed detection.

During the last decade, a lot of efforts and research have been conducted to improve the RAIM and FDE algorithms. Tsai [94] proposed a moving average filter to perform satellite failure detection and exclusion by operating on residuals, while Wang [95] proposed an improved particle filter using a genetic algorithm to compute the weights. Combining different navigation systems, for instance the GPS, GLONASS, Galileo and Beidou, the user can obtain a reliable position solution by applying RAIM/FDE robust algorithms. These types of data integrity are very economical compared to augmentation systems and I believe they will dominate in the future to provide the GNSS integrity solution. That is, with a lot of satellites in view, the user can discard the ones with bad data and still preserve a good geometry in solution computation. Three RAIM methods have been proposed to implement GPS data integrity. These are:

- Range comparison method proposed by Lee [93].
- Least-square residual method proposed by Parkinson and Axelrad [92].
- Parity method proposed by Sturza [91].

For simplicity, we introduce the functionality of the RAIM algorithm based on least-square residuals for illustration. Assume that a number n of satellites in view and the error sources are uncorrelated between the different line-of-sight. One starts out by linearizing the measurements, and the model is given by Eq. (3.62). The linearization process take place around the estimate minus actual position deviation vector β , which is 4-dimensional (one temporal and three spatial coordinates).

The error along each satellite line-of-sight is taken to be zero-mean Gaussian noise with finite variance σ_i^2 . The resulting position estimator can be

expressed as:

$$\hat{\mathbf{x}} = (\mathbf{H}^T \mathbf{W} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{W} \mathbf{z} = \mathbf{P} \mathbf{z} \quad (3.116)$$

where $\mathbf{P}$ is the weighted pseudo-inverse of the design matrix $\mathbf{H}$, and $\mathbf{W}$ is the precision matrix, defined as the inverse of the measurement noise covariance matrix Σ . Often $\Sigma^{-1} = \mathbf{W}$ is referred to as the weighting matrix and is a diagonal $n \times n$ matrix.

LPV-200 is the level of service based on the most demanding flight operation delivered by the WAAS system. The Federal Aviation Administration (FAA) navigation requirement for LVP-200 approaches (150 sec) set the probability α in Eq. (3.115) to be 10^{-7} . Since the vertical component x_v is critical for aviation, the computation of VPL confidence interval reads:

$$\text{VPL} = x_v \pm k(\alpha) \cdot \sigma_v \quad \sigma_v^2 = \left[(\mathbf{H}^T \mathbf{W} \mathbf{H})^{-1} \right]_{3,3} \quad (3.117)$$

$k(\alpha) \sim 5.33$ for integrity risk value $\alpha = 10^{-7}$

NMA Products User Interface

“I hated every minute of training, but I said, ‘Don’t quit’. Suffer now and live the rest of your life as a champion.”

— Muhammad Ali

This chapter introduces the GNSS data processing tools and the basic mathematical background applied in GNSS data processing. This includes the parameter estimation techniques, modeling in space and time, forecasting, prediction, and hypothesis testing. The output from different algorithms will be the GNSS quality indicators that can be used to predict the user data integrity. Figure 4.1 shows all main components needed to

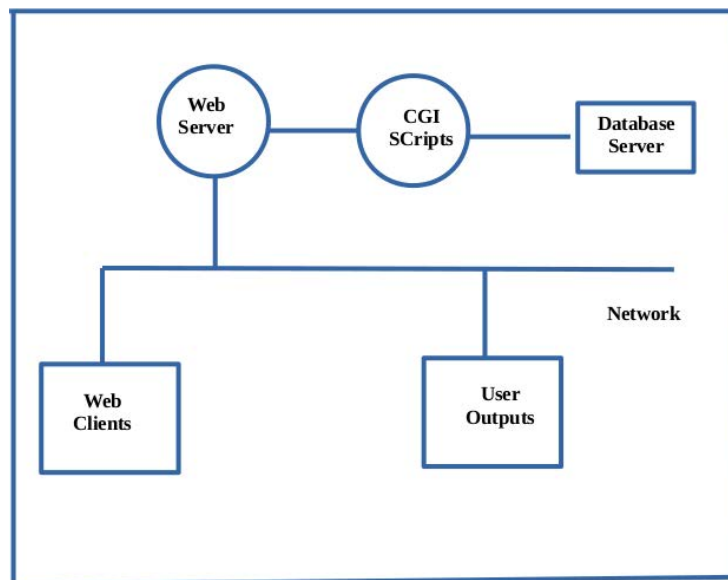


Figure 4.1.: User interface high level functional decomposition

provide GNSS quality indicators to the users. This includes:

- Web server that accept requests from the users and enabling the right applications to process the input and generate the outputs.
- Web client to access the main page and send the client request to the web server.
- Common Gate Interface (CGI) scripts are applications that process the client requests. This includes applications that compute the GNSS

quality indicators and update the database. Other tools as CGI can be considered.

- Database shall store all information processed by the applications.

The user enters the parameters by filling a form by specifying the type of quality indicators (SIS, Atmosphere or local) based on selected GNSS navigation system. The request will be processed by the web server and generate output(s) in form of a simple statistical values, time-series or region with a smooth results. In case of errors or unavailable information, the user will be informed. All request will logged in the database.

Conclusion and recommendations

This chapter summarizes the work done so far and gives some recommendations for future work. Algorithms needed to compute the GNSS performance indicators are given and can be classified:

- Signal-In-Space quality indicators: These are I-URE and GA-URE.
- Atmospheric Quality indicators: These includes ROTI, amplitude scintillation (S_4), phase scintillation (σ_ϕ), I_{95} , IRIU, IRIM, and GRIM.
- Local environment quality indicators: These indicators are algorithms that can be implemented at the user side to reduce the impact of multipath, detecting jamming, spoofing and scintillation. Weighted algorithms such as Danish weighting algorithm, RAIM, and detection algorithms in general can be used for this purpose.

The missing pieces are the implementation and the derivation of all performance indicators for each navigation systems. This shall include at least the GPS, GLONASS, BeiDou and Galileo. These performance indicators shall be stored in database and shall be retrieved by different applications. Finally, the Fault Tree Analysis (FTA) shall be build on the top of all derived performance indicators, and will serve as input to data integrity derivation for different applications.

The derived quality indicators shall be easily accessible by the users (external or internal). A graphical user interface, web based with different options are suggested as implementation solution. These tasks are the aim of the chapter 4, that still under construction.

Procedures of *Ground approach* to compute I-URE for validation shall be added.

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Appendices

A.1. C-ITS og automatisert transport

Kartverket Strategiske handlingsplan 2018-2021.

Kartverket skal hele tiden være i takt med tiden. Alt er i rask endring og vår visjon er at vi skal være med å skape det nye landskapet til enhver tid.

ITS Norge

- Kunnskapsformidling til eksisterende medlemmer for å bidra til å utvikle industrien
- Øke myndighetenes forståelse for ITS på dette området
- Bindeleddene mellom teknologi og næringslivet
- Bindeledd mellom standardiseringskomiteene for GIS og ITS.

Konklusjon

Kartverket, Statens Vegvesen og ITS Norge hadde følgende felles målsetninger for dette forprosjektet:

- Forstå framtidige nøyaktighets- og integritetsbehov for posisjonsbestemmelse av (semi-)automatiserte kjøretøy
- Gjennom testaktiviteter etablere faktagrunnlag for innsikt i kapasiteten til GNSS. for posisjonsbestemmelse/navigasjon langs veinettet, og hvordan teknologien evt. må tilpasses i tjenester og samvirke med andre sensorer for best anvendelse i Norge.

A.2. WP-5 Analyse av ionosfærehendelsene rundt 08.09.2017

The objective of this report is to provide a simple performance assessment of EGNOS for maritime use at high northern latitudes. The assessment is based on data collected from a vessel traveling from 60.4°N to 71.1°N along the Norwegian coast. The data sample is taken from one round-trip, lasting 11 days.

A.3. WP-3.1 Galileo OS Navigation Performance Report

The main objective of this report is to compute the following Key Performance Parameters (KPI) of the Galileo OS in Norway. This includes:

- Signal-In-Space ranging accuracy per satellite and for the entire constellation: E1, E1E5a and E1E5b
- Horizontal positioning service accuracy (HPE95%) per station: E1, E1E5a and E1E5b
- Vertical positioning service accuracy (VPE95%) per station: E1, E1E5a and E1E5b
- 3D velocity service accuracy per station (3D VE95%)

A.4. WP-2.1.2 EGNOS Performance Monitoring in maritime and inland waterways (IWW) domains

The scope of this activity is the provision of technical assistance to GSA for EGNOS performance monitoring in maritime and inland waterways (IWW) domains.

In 2016, European Commission requested GSA to study a new service for maritime based on the EGNOS version 2 and taking into account IMO requirements. GSA with the support of ESSP started the definition of such a potential service, defining the performance parameters in line with IMO resolution A.1046 and starting a preliminary performance analysis. The result of the analysis showed that the most stringent requirement defined by IMO is the continuity, which affects strongly the service coverage area for a potential commitment.

- to validate the service coverage along Norway coast and to correlate events with ionospheric activity
- to compare the results with IALA DGNSS performance.

A.5. EGNOS Performance Monitoring

Two quarterly reports about EGNOS performance are delivered to GSA co-funded project SPMS and to NSA.

A.5.1. EGNOS Performance Report

This report presents a quarterly overview of the performance of EGNOS in Norway as experienced by NMA's monitor stations. The result part of the report contains result data on:

- Availability and continuity of EGNOS SoL service for APV-1 and LPV200 requirements.
- Integrity in the user position domain of the EGNOS SoL service (KPIs: Number of MI epochs, daily maximum HSI, daily maximum VSI).
- Position accuracy (KPIs: HPE95%, VPE95%) of EGNOS and position accuracy of GPS (KPIs: HPE95%, VPE95%) without EGNOS. Both are based on processing the raw data using the same processing strategy.

A.5.2. EGNOS ionosphere model monitoring report

This report details the EGNOS ionosphere model behavior by comparing the values in the broadcast EGNOS ionosphere grid with observations from receivers. The observing network covers the high-latitude parts of the EGNOS grid, and partially covers the mid-latitude parts.

These checks enable a detailed comparison of the EGNOS ionosphere corrections (GIVD) and integrity bounding (GIVE) in the user domain (i.e. projected to the receiver-satellite paths).

A.6. CPOS Network Station data analysis

The main objective of this quarterly quality report is to analyze the GNSS raw data collected at CPOS reference receivers and reports all observed anomalies. This task can be regarded as CPOS Network Station data integrity (ensuring station stability).

A.7. GPS Subsidence data analysis of ConocoPhillips and AkerBP platforms

Purpose: The Norwegian Mapping Authority (NMA) is contracted by Equinor, ConocoPhillips and AkerBP to monitor more than 20 offshore platforms in the North Sea and the coast of Brazil using GPS.

Motivation: Offshore platforms experience subsidence up to decimeter-level per year. These movements are monitored due to safety reasons and to observe the reservoir behavior from deformations of the solid Earth's surface (seabed). The safety reasons includes short term incidents, e.g. earthquakes and vessel collision and maintenance of sufficient air gap between the platform and the sea surface in case of storm waves.

Analysis: GPS site coordinates are computed using the GIPSY-OASIS software by applying PPP. GPS data quality is analyzed via time series from daily post-fit residuals, number of observations and outliers for pseudo-range and carrier phase observations. These data quality indicators monitor the performance of GPS equipment, receiver configuration changes and local environment conditions. In addition GPS time series are generated from the daily site coordinates to determine linear and harmonic rates, which gives an indication about platform subsidence.

Publication: A continuous velocity field for Norway [6].
The main objective of this article is to construct the velocity field for the entire mainland of Norway using the GNSS data for the first time.

A.8. GNSS signal degradation in difficult environment

Purpose: I denne studien har vi, basert på data samlet inn på en trikk i ordinær rutetrafikk i Oslo, sett på det europeiske satellittnavigasjonssystemet Galileo som et tillegg til det amerikanske satellittnavigasjonssystemet GPS og målt systemytelse i form av tilgjengelighet til posisjon (uttrykt som prosentandel av tiden hvor brukerens mottaker er i stand til å beregne en posisjon), og posisjonsnøyaktighet (uttrykt gjennom horisontal og vertikal posisjonsfeil).

Det er velkjent at bygninger og naturlige hindringer reduserer synligheten til satellitter og legger til forskjellige slags feil i avstandsmålingene fra satellittene på grunn av reflekser fra omgivelser. Dette kan produsere en posisjonsfeil på ti til hundrevis av meter. Av resultatene observerer vi at

1. Bruk av Galileo som et tillegg til GPS gir bedre tilgjengelighet og nøyaktighet for brukere på bakken. Dette er spesielt tydelig i områder med høye bygninger.
2. Galileo gir en noe bedre nøyaktighet enn GPS.
3. Nøyaktigheten til GPS+GAL er bedre enn GPS alene.

Motivation: Denne rapporten gir et kort sammendrag av arbeidet med å analysere ytelsen til Galileo i bystrøk (Oslo). Arbeidet er en del av et prosjekt som er initiert av Norsk Romsenter og som i skrivende stund fremdeles pågår. Dataene som ligger til grunn for analysen er GNSS-rådata som har blitt logget med en multi-GNSS- mottaker, i dette tilfellet, systemene GPS, GLONASS, Galileo og EGNOS. Mottakeren er montert på en trikk, og den måler kontinuerlig, både når trikken er i bevegelse og i ro. Dataene lagres i mottakeren. Rådataene har blitt brukt til å beregne mottakerens posisjon. Disse posisjonene har blitt sammenlignet med en fasit som gir tilnærmet riktig posisjon av trikkeskinnene. Trikken kjører alltid på samme spor, men mottakeren er montert på siden av trikken slik at det er et lite avvik i forhold til trikkesporene og trikkens midtpunkt og dermed også i forhold til retningen som trikken kjører. Det har blitt valgt ut tre forskjellige områder som representerer åpent terreng (Åpent område), tettbebyggelse (Tettbygd) og høyhus-bebyggelse (Sentrumsbebyggelse).

Analysis: Resultatene fra dette studiet kan vi trekke to hovedkonklusjoner:

1. Tilgjengeligheten til posisjon for brukere på bakken blir bedre ved å bruke Galileo-satellitter i tillegg til GPS-satellitter. Dette er spesielt tydelig i områder med høye bygninger.
2. Galileo gir en noe bedre posisjonsnøyaktighet enn GPS.

Analysen gir ikke et fullstendig bilde av ytelsen til Galileo slik den vil bli når alt er på plass og systemet er fullt utbygd. I første rekke viser den at Galileo gir færre beregnede posisjoner enn GPS. Dette skyldes to ting:

1. At Galileo ikke er ferdig utbygd. Den har 22 satellitter mot GPS sine 31.
2. At GNSS mottakeren i testperioden ikke var i stand til å levere observasjoner fra 3 av de nyeste Galileo-satellittene. Dette medførte at totalt antall Galileo-satellitter i studien ble 19 istedenfor 22, og følgelig at tilgjengeligheten til posisjon ved bruk av Galileo alene ble dårligere enn det reelt sett er.

Det må derfor forventes at ytelsen til Galileo vil bli ytterligere forbedret når systemet er komplett.

Publication: NONE

A.9. Space Weather impact on Arctic Navigation (SWAN)

Purpose: The main objective of this activity is to develop and demonstrate new techniques, which are expected to improve on the current state-of-the-art Space Weather forecasting techniques and products related to Space Weather effects for Positioning, Navigation and Timing (PNT) systems in the Arctic, with particular attention to the Greenland area. For this purpose, the activity will investigate and test innovative methods for monitoring and predicting Space Weather impacts on PNT in that area. The activity will be based on an experimentation testbed, in which new or enhanced forecast Space Weather products can be derived. Detailed description of the project is given by [1].

A.10. Space weather impact on GNSS and development of GNSS performance indicators (P3-SWE-XLLI)

Purpose: the overall objective of the work is to develop and operate a user-tailored GNSS performance indicator application providing information on the expected impact of space weather on positioning at the user's location which has been identified as high-priority development in order to meet growing end user needs for space weather information in this domain. A concept and initial implementation has already been developed with the context of the ionospheric Weather Expert Service Centre. Detailed description of the project is given by [2].

A.11. NSIA Project

Purpose: Development of autonomous capabilities across safety-critical applications such as autonomous operation of unmanned cars, surface vessels (e.g. unmanned passenger ferries) and aerial vehicles (e.g. air-taxis and air ambulances) is increasing the demand for navigation

system integrity and accuracy. Although autonomous system development is an area undergoing extensive research worldwide, the available technology is not yet mature enough to support fully autonomous safety critical operations. Simultaneously, navigation accuracy and safety requirements for such system operations are also at an early stage of definition. This document is an internal NSIA project deliverable D1 (WP2- System Concept and User Requirements). The purpose of it is to summarize existing and anticipated user requirements for navigation system performance in support of safety critical autonomous operations considering three sectors: road, aerial, marine and potentially the railway.

Detailed description of the project is given by [3].

A.12. Station location definition procedures

Procedures and directives on how to choose the station location are given by [27].

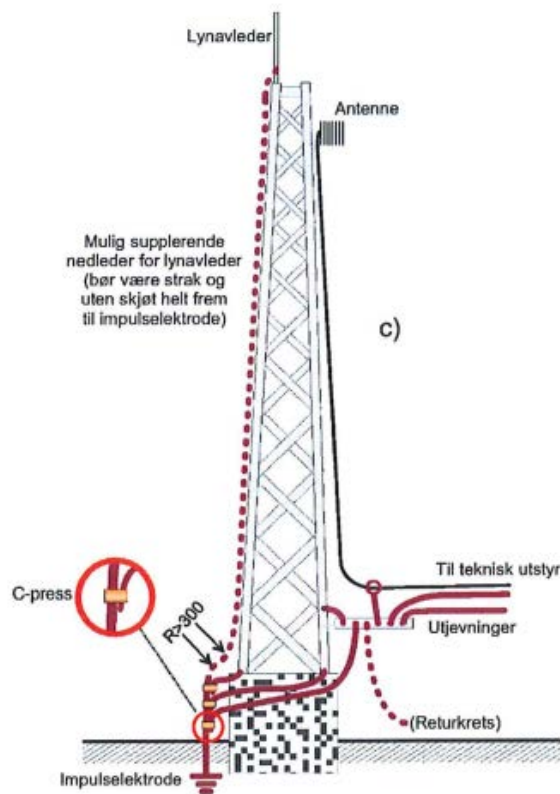


Figure A.1.: Grounding and surge protection against electric shock hazard



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